Reputation and Managerial Truth-Telling as Self-Insurance

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1. Introduction

Corporate insiders such as managers and analysts often provide earnings forecasts to investors. These predictions can increase capital market efficiency, but also offer the reporter an opportunity for private gain. Thus, even when the quality of information possessed by a forecaster is common knowledge, reputation for honesty is critical.

Previous literature depicts truth-telling as, at least for some agents, an inherent characteristic. For example, Sobel (1985) and Benabou and Laroque (1992) assume that one type of agent is always honest. Our paper offers a model in which honesty is endogenous for all types,[1] and the depiction of truth-telling that emerges is more cynical. Agents report accurately when their current circumstances are good and reputation-building is affordable; agents lie when their current circumstances are dire and they require income. We thereby introduce a self-insurance role for reputation to the literature.

We consider an infinite horizon setting where a single risk-averse agent obtains noisy signals about the end of period cash flows of a firm. The corporate insider makes forecasts to a market of competitive risk-neutral investors. The investors then rationally update the value of the firm using the history of relevant cash flows and forecasts, and the manager is paid a wage. The insider cannot affect future cash flows, and the quality of the signals he receives are common knowledge. We can view this insider as a corporate manager or security analyst,[2] although our focus is on the former.

The economic mechanism by which our manager benefits from his informational monopoly is unique in the literature. Previous work has considered insiders who trade on private information (Kyle, 1985) and insiders who sell private information (Admati and Pfleiderer, 1986). We restrict our manager from undertaking either of these activities, and instead expose the manager to stochastic, persistent, and externally unobservable consumption shocks. We call this "non-wage income," but it could be anything that changes the marginal benefit of wage income over time. For example, such shocks could reflect the level of living expenses the agent must incur each period, or relate to non-monetary exogenous events that alter the marginal utility of consumption. We assume that the manager has imperfect ability to hedge non-wage income risk through financial markets, due to either incomplete markets or the costs of hedging. We then consider wage contracts of different forms that can depend on variables such as firm value or

[1] Morris (2001) analyzes purely endogenous reputation in a different setting where a "good" political advisor would not like to be confused with an intrinsically biased advisor. Focusing on reputation for quality rather than honesty, Tadelis (1999) and Mailath and Samuelson (2001) focus on the "market for reputation" when producers of heterogeneous quality may unobservably sell their brand name to others.

[2] Models of analyst behavior often focus on other issues such as strategic interactions between multiple analysts and learning about analyst quality. Examples include Scharfstein and Stein (1990), Trueman (1994), Zweibel (1995), Prendergast and Stole (1996), Avery and Chevalier (1999), Graham (1999), and Ottaviani and Sorensen (2005).

forecast accuracy.

The most interesting equilibrium occurs when the manager's compensation increases with firm value. Because of his desire to smooth consumption, the manager uses selective truth-telling about his private information to manipulate the stock price and hence his wage. The manager truthfully reveals his signal and builds reputation when non-wage income is high, but overstates his signal on average when non-wage income is low. This outcome, which we call the *self-insurance equilibrium*, formalizes the idea that reputation may be built up in good times so that it can be exploited in times of need.

Reputation in the self-insurance equilibrium is endogenous. Specifically, reputation is the rationally updated probability that the manager can afford honesty because non-wage income is high. A low earnings forecast gives instant credibility because it signals that the manager has high non-wage income. After subsequent high earnings forecasts, reputation dwindles: Investors cannot be certain if the signal is actually high, or if the manager has low non-wage income and thus cannot afford honesty. A positive forecast following a string of positive forecasts (when reputation has deteriorated) has less price impact than if it immediately follows a low forecast (when reputation is highest). Consistent with this prediction, Stickel (1992) finds that the price impact of a positive analyst report depends on the reputation of the analyst, but that negative reports have price impact independent of prior reputation. Our self-insurance equilibrium suggests that this may occur because a negative report gives instant credibility and instant reputation.

The compensation schemes we analyze are not meant to provide an exhaustive analysis of possible payoff structures. Rather, we focus on three basic types that are easy to implement and occur frequently in the real world. In *flat-wage* compensation, the same amount is paid to the manager in each period regardless of observed state variables. *Accuracy-based compensation* increases with the realized precision of the current period cash flow forecast. Finally, *value-based compensation* increases with the current period stock price of the firm. In practice, it seems likely that accuracy is an important component of analyst compensation.[3] For corporate executives, an extensive empirical literature shows that flat-wage and value-based compensation are the most common.[4]

All three of the above compensation schemes are static, and hence do not require complex long-term contracts. Further, pay is based on variables that are easy to observe and practical to contract on. We explore more complicated employment arrangements by using the revelation principle to infer a dynamic contract that gives the same degree of

[3]Recent literature suggests that analysts may also have some component of their compensation that increases in firm value because of pressure to satisfy current and potential investment banking clients. See, for example, Schipper (1991), Francis and Philbrick (1993), Michaely and Womack (1999), Lim (2001), and Hong and Kubik (2003).

[4]See, for example, Aggarwal and Samwick (1999), Garvey and Milbourne (2004), and Ortiz-Molina (2005).

consumption smoothing as the self-insurance equilibrium, but achieves full informational efficiency. The disadvantage of the equilibrium inferred from the revelation principle is that pay must then depend on the entire history of reports and cash flows, and long-term employment contracts are then necessary. In the much simpler self-insurance equilibrium, endogenous reputation is a sufficient statistic for the history of relevant state variables. Further, because managerial reputation is fully impounded into firm value, the self-insurance equilibrium permits a static contract in the observable and easily contractible stock price.

The three basic types of equilibria that we identify are pooling (or babbling) outcomes in which no information is conveyed by the manager; a semi-separating equilibrium in which the manager sometimes lies and sometimes tells the truth (the self-insurance equilibrium); and a separating equilibrium in which the manager's forecast is always accurate. From a welfare perspective, the tradeoffs between these equilibria require consideration of informational efficiency and the consumption smoothing motive of the manager. Of course a regulator may consider only informational efficiency to be relevant, in which case the flat-wage or accuracy-based compensation should be chosen at the cost to investors of a higher average salary to the insider. We emphasize that, even under value-based compensation where there are no apparent pay incentives for accuracy, some degree of honesty is induced due to endogenous concern for reputation.

Before we solve the infinite-horizon model with value-based compensation, we analyze a single-period model in which managers have fixed type. Analogous to models where reputation can be sold, we assume an exogenous linear utility benefit to terminal reputation in the single-period model. The infinite-horizon model then extends the analysis to achieve an endogenous derived utility benefit to reputation. Instead of the market for reputation that has been developed in previous literature, (e.g., Tadelis, 1999; Mailath and Samuelson, 2001), the endogenous benefit of reputation stems from the self-insurance motive of our manager.

A caveat in interpreting our model is that in equilibrium, all agents play their strategies exactly as expected by other agents. Hence, notions such as "honesty" and "lying" are clearly unrelated to whether an agent maintains or deviates from his equilibrium strategy. Instead, "truth-telling" in the context of our model should be understood from the reference point of a naive outside observer who equates honesty with accurately reporting the cash flow signal.[5] Prior literature similarly conceptualizes "honesty" and "lying" in the context of equilibrium play.[6]

[5]A sophisticated observer of the game will understand that the insider's message communicates information about both his cash flow signal and non-wage income. A naive observer may interpret the manager's message only as a forecast of future cash flows, and it is from this perspective that notions of honesty and truth-telling are best understood.

[6]See, for example, Benabou and Laroque (1992), Morris (2001), and Sobel (1985) discussing "honesty," "truthfulness," and "lies" in the context of equilibrium play.

Section 2 sets out the infinite horizon model. Section 3 studies equilibrium under flat-wage and accuracy-based compensation. Section 4 analyzes value-based compensation and the self-insurance equilibrium. Section 5 considers social welfare. Section 6 uses the revelation principle to derive a dynamic contract with an expanded message space, and examines its properties. Section 7 concludes. All proofs are in Appendix C unless stated otherwise.

2. An Infinite-Horizon Cheap-Talk Message Game

This section sets up a discrete time, infinite horizon model of managerial reporting. The model is a standard cheap-talk message game (Crawford and Sobel, 1982; Sobel, 1985), in which a sender (the manager) observes the state of nature and conveys a message to a receiver, who then takes an action that determines payoffs. As in Benabou and Laroque (1992), the model involves multiple receivers (investors), and their aggregate reaction determines the market clearing price for shares in the firm. The manager's payoff in each period is a wage that can depend on current stock price or forecast accuracy.

The information received by the manager is a constant-quality signal of the future firm cash flow. Investors use the manager's forecast and the subsequently observed cash flow to update their beliefs about the current state of nature. We characterize equilibria in subsequent sections.

2.1. Outline of the Model

At the beginning of each period $t = 1, 2, ...$, nature draws a state (N_t, S_t) that is privately observed by the manager, but not directly observed by investors. The variable $N_t \in \{N^h, N^l\}$, where $N^h > N^l$, corresponds to the level of non-wage income the manager will receive in the period. Non-wage income evolves stochastically according to a first-order Markov switching process with state transition matrix

$$Q \equiv \begin{bmatrix} q^{hh} & q^{hl} \\ q^{lh} & q^{ll} \end{bmatrix},$$

where $q^{ij} \equiv \mathbb{P}\left(N_t = N^j | N_{t-1} = N^i\right)$, for $i, j \in \{h, l\}$ and $t > 0$. For parsimony we denote $q \equiv q^{hh} = q^{ll}$, and assume $q > .5$ so that non-wage income is persistent.

The signal $S_t \in \{S^h, S^l\}$, where $S^h > S^l$, gives imperfect but private information to the manager about an end-of-period cash flow. The probability of receiving the high signal is constant over time,

$$p \equiv \mathbb{P}(S_t = S^h).$$

The draws of S_t in each period are independent of one another and of the prior play of the game.

After privately observing the state (N_t, S_t), the manager issues a forecast to investors of end-of-period earnings. This forecast is given in a report R_t. Most of the equilibria we discuss use only two messages. For notational convenience we thus begin by restricting the message space to the values $R_t \in \{R^h, R^l\}$.[7] We assume for simplicity that the firm is financed entirely by a single infinitely divisible equity share. Once the report is issued, investors update their beliefs about the value of the firm, and establish a new market price V_t for the equity.

At the end of each period, the firm stochastically realizes cash flows $C_t \in \{C^h, C^l\}$ with $C^h > C^l$. For $i, j \in \{h, l\}$, denote the conditional probability of the end-of-period cash flow given the beginning-of-period signal by

$$\lambda^{ij} \equiv \mathbb{P}\left(C_t = C^j | S_t = S^i\right)$$

for $t > 0$. We reduce the number of parameters by assuming that both signals are of equal quality $\lambda \equiv \lambda^{hh} = \lambda^{ll} > 0.5$. This specification permits a noisy signal $(\lambda < 1)$, as in Benabou and Laroque (1992), as well as the special case of a perfectly informative signal $(\lambda = 1)$. For $i \in \{h, l\}$ and $j \neq i$, we also let

$$S^i = \mathbb{E}\left(C|S^i\right) = \lambda^{ii} C^i + \lambda^{ij} C^j.$$

Thus, the signal S is itself the conditional expectation of the future cash flow C, which requires $C^l \leq S^l < S^h \leq C^h$. The vector (S_t, C_t) is iid over time with a joint distribution characterized by the parameters p, λ, and equivalently either $\left(C^l, C^h\right)$ or $\left(S^l, S^h\right)$. Simultaneously with the realization of cash flows, wages W_t are paid to the manager, and shareholders claim the residual as a dividend.

To initialize the game at $t = 0$, we let nature draw $N_0 \in \{N^h, N^l\}$ with probabilities q_0^h and q_0^l respectively. Note that if $q_0^h = q_0^l = 0.5$, then N_0 is drawn from the steady state distribution of N_t,[8] but allowing $0 \leq q_0^h \leq 1$ permits more general initializations as well.

2.2. Investor Information and Beliefs

Investors begin the game at $t = 0$ knowing the stochastic structure of the game and all parameters, but do not know the realization of N_0. We denote their initial information set by Φ_0.

For $t > 0$, we recursively define $\Omega_t \equiv \{\Phi_{t-1}, R_t\}$ and $\Phi_t \equiv \{\Omega_t, C_t\}$. Thus, $\Omega_t \subseteq \Phi_t$ denotes investor information after receiving the manager's report R_t, but before

[7] In the final section of the paper, we use the revelation principle to infer an equilibrium that uses four messages, but requires complicated dynamic compensation contracts.

[8] It is straightforward to observe that since the transition matrix Q is symmetric, the steady state distribution of N_t puts equal weights on N^h and N^l.

observing the cash flow C_t. The end-of-period information set Φ_t includes the cash flow realization.

Assuming Bayesian updating, these information sets can be used to derive investor beliefs about the manager's type N_t at various stages of the game. For $i \in \{h, l\}$, define $\Pi_t^i \equiv \mathbb{P}\left(N_t = N^i \middle| \Omega_t\right)$ and $\Psi_t^i \equiv \mathbb{P}\left(N_t = N^i \middle| \Phi_t\right)$. Also let $\Pi_t \equiv \left(\Pi_t^h, \Pi_t^l\right) \in \mathbb{R}_+^2$ and $\Psi_t \equiv \left(\Psi_t^h, \Psi_t^l\right) \in \mathbb{R}_+^2$. Initial beliefs satisfy $\Psi_0^h \equiv \mathbb{P}\left(N_0 = N^h \middle| \Phi_0\right) = q_0^h$. For $t > 0$ and $i \in \{h, l\}$, beliefs can be updated recursively using

$$\Pi_t^i = \mathbb{P}\left(N_t = N^i \middle| \Psi_{t-1}, R_t\right) \tag{2.1}$$

$$\Psi_t^i = \mathbb{P}\left(N_t = N^i \middle| \Pi_t, C_t\right). \tag{2.2}$$

These equations implicitly define updating functions $\Pi_t = \Pi(\Psi_{t-1}, R_t)$ and $\Psi_t = \Psi(\Pi_t, C_t)$. Embedded within these functions is the reporting strategy that investors suppose the manager is using.

2.3. Managerial Preferences and Wages

The manager has utility $U_t = \sum_{s=0}^{\infty} \beta^s u(c_{t+s})$, where c denotes consumption, $0 < \beta < 1$ is the subjective discount rate, and $u(c)$ is a risk-averse utility function. The manager consumes all wage and non-wage income each period, giving

$$c_t = N_t + W_t. \tag{2.3}$$

We consider three static compensation schemes: flat-wage, accuracy-based, and value-based. Functional form details are provided in subsequent sections, and we discuss here the motivation for choosing these compensation types.

The flat-wage and accuracy-based compensation provide useful benchmarks. These wage structures provide no incentive for misrepresenting the true state, and so fully revealing equilibria are a natural outcome. It also seems likely that the flat-wage and accuracy-based compensation are important in practice. For example, Garvey and Milbourn (2005) document that about 16% of total CEO compensation from 1992-2001 was in the form of base salary.[9] While there is no direct empirical evidence of accuracy-based compensation for corporate executives, it would seem to be especially important for security analysts, whose primary function is to forecast corporate outcomes.[10]

The main result of our paper is the self-insurance equilibrium, derived from value-based compensation. The empirical literature documents a large proportion of executive pay with high sensitivity to underlying stock price performance. For example, Garvey

[9] Using different samples, Aggarwal and Samwick (1999) and Ortiz-Molina (2005) report that base pay constitutes 25% and 40% of total compensation respectively.

[10] Recent literature suggests that indirect incentives to satisfy investment banking clients may also give security analysts some degree of value-based compensation.

and Milbourn (2005) find that from 1992-2001, 64% of total CEO compensation was in the form of stock option grants, and another 20% in the form of other short-term performance based incentives. With different samples, Aggarwal and Samwick (1999) document 43% stock-based compensation and another 22% in short-term bonuses, and Ortiz-Molina (2005) records approximately 30% for each category. Value-based compensation is thus empirically important for corporate managers.

The empirical prevalence of stock-based compensation for corporate executives is not surprising, given the strong theoretical motivation for aligning managerial incentives in order to help overcome moral hazard problems (e.g., Ross, 1973; Jensen and Meckling, 1976; Holmstrom, 1979). In our paper, our concern is not to model the well-understood motivation for value-based compensation, but rather to explore the implications of this type of compensation for managerial truth-telling.

2.4. Valuation

Investors in this model act as a pricing mechanism. We follow Kyle (1985) and use the *market efficiency condition*:

$$V_t = \mathbb{E}\left[\sum_{s=0}^{\infty} \beta^s \left(C_{t+s} - W_{t+s}\right) \middle| \Omega_t\right].$$

As noted by Kyle, the market efficiency condition can be replaced by assuming $n \geq 2$ risk-neutral market makers who are Bertrand price competitors.[11]

We can decompose equity value V_t into the present value V_t^* of gross cash flows and the present value W_t^* of wages:

$$V_t = V_t^* - W_t^*. \tag{2.4}$$

We then observe that

$$V_t^* = \mathbb{E}\left(C_t | \Omega_t\right) + k, \tag{2.5}$$

where $k = \frac{\beta}{1-\beta}\mathbb{E}\left(C\right)$.The first component is the conditional expectation of the current cash flow, about which investors may have some information through the report of the manager. The second component k is the perpetuity value of cash flows from $t+1$ on. Neither the manager nor investors have any information about these future flows beyond their common knowledge unconditional distribution.

[11] Bertrand price competition among market makers is used explicitly in a variety of micro-structure settings by Bagnoli, Viswanathan, and Holden (2001). Under value-based compensation, a caveat is that a lower share price reduces wages and hence increases investors' share of future cash flows. An individual would thus want to sell a small part of his holdings at an artificially low price to increase dividends on his remaining holdings. This possibility is easily eliminated by requiring that at least two market-makers always have no shareholdings. Alternatively, if market-makers must stand ready to sell any and all holdings at their quoted price, there would also be no incentive to underprice.

2.5. Markov Strategies and Markov Perfect Bayesian Equilibrium

Given the stationary Markov structure of the game, it is natural to restrict attention to Markov strategies. At any point in time t, the non-wage income N_t, the signal S_t, and the prior beliefs $\Psi_{t-1} \in [0,1]$ completely summarize the relevant history of the game. The manager then has pure strategies $\left\{R : (N_t, S_t, \Psi_{t-1}) \to \left(R^h, R^l\right)\right\}$. Taking probability measures over these gives the Markov strategy space $\mathcal{R} = \{\rho : (N_t, S_t, \Psi_{t-1}) \to [0,1]\}$. A strategy $\rho \in \mathcal{R}$ thus defines for $k \in \{h, l\}$ the conditional declaration probabilities:

$$\rho_k^{ij}(\Psi) \equiv \mathbb{P}\left(R_t = R^k \middle| N_t = N^i, S_t = S^j, \Psi_{t-1} = \Psi\right),$$

which sum to one for all $i, j \in \{h, l\}$ and $\Psi \in [0,1]$. Since R^h and R^l are arbitrary labels, we assume without loss of generality that $\rho_h^{lh}(\Psi) \geq 0.5$ for all $\Psi \in [0,1]$. Given a strategy $\rho \in \mathcal{R}$ for the manager, investor beliefs are also Markov due to the recursive nature of the updating. We then consider *Markov perfect Bayesian equilibria*, characterized by 1) The manager's strategy ρ is maximizing for all (N_t, S_t, Ψ_{t-1}), 2) Firm value is given by (2.5), and 3) Investors update beliefs using Bayes' rule whenever possible.[12]

Another way to state the equilibrium condition is that the strategy ρ in every state maximizes the Bellman Equation

$$J(N_t, S_t, \Psi_{t-1}) = \max_{\rho \in \mathcal{R}} \mathbb{E}\left\{u(c_t) + \beta J(N_{t+1}, S_{t+1}, \Psi_t) \,|N_t, S_t, \Psi_{t-1}\right\}, \qquad (2.6)$$

subject to investors updating using (2.1) and (2.2) and embedding the strategy ρ in expectations, firm valuation given by (2.5), wages as specified in the subsections below, and consumption constrained by (2.3).

3. Flat-Wage and Accuracy-Based Compensation

This section describes the equilibrium outcomes associated with flat-wage and accuracy-based compensation. We assume that the manager has pay of the form

$$W_t = a + b\mathbf{1}_{\{R_t = C_t\}},$$

where $b \geq 0$, and for any event A the indicator function $\mathbf{1}_{\{A\}}$ takes the value one when A is true and zero when A is false. If $b = 0$, the compensation is a flat-wage, and $b > 0$ corresponds to accuracy-based compensation. We can then show

Proposition 1. *With accuracy-based or flat wage compensation, any fully revealing strategy is an equilibrium strategy. In such an equilibrium, investors set firm value believing that the manager is truthfully revealing his signal. With flat-wage compensation,*

[12] For any Ψ_{t-1}, as long as the manager uses each report in some (N, S) state of the world with positive probability, then conditional expectations using Bayes' rule are always well-defined. This is further explained in Appendix A.

uninformative pooling equilibria also exist. In these equilibria, investors infer nothing about the manager's type or cash flow signal from his message.

In both the full revelation (FR) and pooling (PO) equilibria, the stock price of the firm has trivial dynamics. In particular, under PO, the share price is constant and always equal to the discounted unconditional expectation of cash flows. The FR stock price is stochastic, but since the manager's signal is fully revealed, the share price is independent of all variables from previous periods. Under the natural assumption that investors benefit from better information, it is clear that a flat wage Pareto dominates any accuracy-based compensation due to the risk aversion of the manager.[13]

4. Value-based Compensation and Reputation

While accuracy-based compensation seems natural for securities analysts, both empirical evidence and theory suggest that value-based compensation should be important for corporate executives. This section develops the implications of value-based compensation for managerial truth-telling. We proceed in two steps. First, we develop a simplified single-period model in which the benefit to "reputation" is exogenous. We then proceed to the generalized infinite-horizon setting, in which reputation benefits are endogenous, and motivated by self-insurance.

4.1. A Single Period Model

In the single period model, investors begin with initial beliefs Ψ_0^h and Ψ_0^l that the manager is, respectively, of the high or low non-wage income type. The row vector $\Psi_0 = \left(\Psi_0^h, \Psi_0^l\right)$ sums to one. As in the infinite horizon model, the manager receives a signal S, taking the value S^h with probability p and value S^l otherwise. The manager plays a strategy $\rho(N,S)$ that gives the probabilities for his choice of message $R \in \left\{R^h, R^l\right\}$ in each state. After observing the report, investors update the value $V(R)$ of the firm and pay a wage

$$W(V) = a + \alpha V(R). \tag{4.1}$$

[13] Although accuracy-based compensation is Pareto dominated by a flat wage in our setup, it is not difficult to motivate settings where a flat wage no longer produces full revelation. For example, an earlier version of this paper discussed the case where signal acquisition is costly to the manager. An incentive is then necessary to induce the manager to incur this cost, and a flat wage is no longer consistent with FR.

The market-efficiency condition gives the value of the firm:[14]

$$V(R) = \frac{1}{1+\alpha}\left[\mathbb{E}(C\,|R) - a\right]. \tag{4.2}$$

At the end of the period, the cash flow C is observed, and investors update their beliefs about the type of the manager.

We exogenously specify "reputation" as investors' ex-post probability that the manager is in state N^h, after observing the report R and cash flow C. Denote these terminal beliefs, for $i \in \{h, l\}$, by

$$\Psi_1^i \equiv \mathbb{P}\left(N^i\,|R, C\right). \tag{4.3}$$

Appendix A shows how to calculate these beliefs.

The manager consumes all of his wage and non-wage income, and has total utility linear in his terminal reputation:

$$U\left(N, W, \Psi_1^h\right) = u(N+W) + \theta\Psi_1^h,$$

where $\theta > 0$. The manager thus solves

$$J(N, S) = \max_{\rho}\left\{u(N+W) + \theta\mathbb{E}\left[\Psi_1^h\,|S\right]\right\} \tag{4.4}$$

subject to wages (4.1), firm valuation (4.2) and the updating of beliefs (4.3).

Since this is a single-period game of incomplete information, the natural solution concept is *Bayes-Nash Equilibrium*. This requires that: 1) The manager maximize his payoff by solving (4.4), 2) Firm value is given by (4.2), and 3) Investors update beliefs by Bayes' rule whenever possible.

We now analyze equilibria of the single-period model. A pooling equilibrium always exists, and a semi-separating, or *partial revelation (PR)*, equilibrium may exist. We call any strategy that conveys no information about the manager's type or signal an *uninformative* strategy. An example is randomly declaring high or low earnings in a manner unrelated to the state (babbling). Any uninformative strategy is an equilibrium strategy.

In the PR equilibrium, the managerial type with high non-wage income N^h reports his signal S truthfully (i.e., $R_t = R^i$ if $S_t = S^i$ for $i = h, l$),[15] while the type with low non-wage income N^l always reports a high signal. The economic foundation of the

[14] As in the infinite horizon model, $V^* = W + V$, decomposes the value of gross firm cash flows, V^*, into management wages and the value of equity. The assumption $W = c + \alpha V(R)$ implies that wages are also linear in the gross value of the firm. Specifically, $W(V^*) = a^* + \alpha^* V^*$, where $\alpha^* = \alpha/(1+\alpha)$ and $a^* = a/(1+\alpha)$. It is straightforward to use the market-efficiency condition to obtain $V^*(R) = \mathbb{E}(C|R)$, which leads to the equation for $V(R)$.

[15] This definition of "truthful" is identical to Benabou and Laroque (1992, p.926), Morris (2001, pp. 239-240), and Sobel (1985, p. 562).

PR equilibrium is the difference in the marginal utilities of the high and low non-wage income types. Because the high non-wage income type has lower marginal utility of present consumption, he is more willing to forego current wages in favor of building future reputation. The low non-wage income type is more likely to forego reputation building in favor of current wages. Hence a semi-separating equilibrium may result.

Let $V^{PR}(R)$ and $\Psi_1^{PR}(R)$ be, respectively, the firm valuation function and the terminal belief updating function of investors who form expectations assuming that the PR strategy is being played. Also define

$$\Delta_u^{PR}\left(N^i\right) \equiv u\left(N^i + \alpha V^{PR}\left(R^h\right) + a\right) - u\left(N^i + \alpha V^{PR}\left(R^l\right) + a\right),$$

which is the utility differential, through wages, from declaring high earnings. Similarly, let

$$\Delta_\Psi^{PR}\left(S^j\right) \equiv \theta \mathbb{E}\left[\Psi_1^{PR,h}\left(R^l\right) - \Psi_1^{PR,h}\left(R^h\right) | S^j\right]$$

be the utility differential, through reputation, from declaring low earnings. From the equations for wages and updated beliefs derived in Appendix A, it is straightforward to observe that $\Delta_u^{PR} \geq 0$ and $\Delta_\Psi^{PR}\left(S^j\right) \geq 0$. We can then succinctly state the necessary and sufficient conditions for the PR strategy to form an equilibrium:

$$\begin{array}{ll} C1: & \Delta_u^{PR}\left(N^h\right) \geq \Delta_\Psi^{PR}\left(S^h\right) \\ C2: & \Delta_u^{PR}\left(N^h\right) \leq \Delta_\Psi^{PR}\left(S^l\right) \\ C3: & \Delta_u^{PR}\left(N^l\right) \geq \Delta_\Psi^{PR}\left(S^l\right). \end{array}$$

More generally,

Proposition 2. *The pure strategy equilibria of the one-period model are of at most two types characterized by*

i) Any uninformative strategy is an equilibrium strategy.

ii) If conditions $C1$ to $C3$ are satisfied, then the PR strategy is an equilibrium strategy.

iii) Other than the PR strategy, no informative pure strategy is an equilibrium strategy.[16]

Proposition 2 gives conditions under which the partial revelation strategy exists, and shows that if it exists, it is the unique informative pure strategy equilibrium.[17]

[16] The proof rules out three and four message strategies in addition to the two message strategies we focus on in this section.

[17] There are also mixed strategy equilibria similar to the PR equilibrium. For example, the manager could announce R^h with probability one in all states except $\left(N^h, S^l\right)$, and be indifferent to the choice of R in this state allowing a mixed strategy. It is also possible that the manager mixes in state $\left(N^h, S^h\right)$ while playing the pure PR strategy elsewhere, or mixes in $\left(N^l, S^l\right)$ while playing the pure PR strategy elsewhere. It is not possible in equilibrium for the manager to mix in state $\left(N^l, S^h\right)$.

The three conditions required are sufficient to ensure that in all four (N, S) states of the world, the manager's action is optimal. Only three conditions are needed because they together ensure that a fourth condition, $\Delta_u^{PR}\left(N^l\right) \geq \Delta_\Psi^{PR}\left(S^h\right)$ is also satisfied. Intuitively, if the manager finds it optimal to announce R^h when the state is $\left(N^h, S^h\right)$, he will announce the same way when the state is $\left(N^l, S^h\right)$, due to concavity of the utility function.

A brief description of when the existence conditions fail helps to give some intuition for the model. First, if pay sensitivity α is too small, condition $C1$ will be violated. Reputational concerns dominate the benefits of current wages, and the manager reports R^l rather than R^h in state $\left(N^h, S^h\right)$. On the other hand, when α is very large condition $C2$ may be violated. In this case, pay sensitivity is too strong relative to the reputation effect, and the manager fails to declare R^l in state $\left(N^h, S^l\right)$.

Similar exercises can be carried out for the other parameters of the model, and all of the instances where the PR equilibrium fails can be traced to an imbalance between reputational concerns and wage incentives in one of the (N, S) states of the world. Conditions $C1$ to $C3$ ensure that such imbalances do not occur. In the infinite horizon model, similar conditions on fixed parameter values will be required to ensure the existence of a PR equilibrium.

4.2. Value-Based Compensation in the Infinite-Horizon Model

The single-period model specifies an exogenous and linear utility reward for terminal reputation. We now endogenize this reward by extending to an infinite horizon setting. The endogenous benefit of reputation is the ability to, when income is needed, overstate cash flow forecasts with greater credibility.

The value-based wage contract is assumed linear in firm value:

$$W_t = a + \alpha V_t \tag{4.5}$$

with fixed coefficients a and α. We then observe

Proposition 3. *If the wage W_t satisfies $W_t = a^* + \alpha^* V_t^*$ for some coefficients a^** and *α^*, then it also satisfies equation* (4.5) *with coefficients a* and *α given the Appendix.*

In the remainder of the paper we set $a^* = -\alpha^* V_{\min}$, where $V_{\min} = \frac{\beta}{1-\beta}\mathbb{E}(C) + S^l$ is the lowest possible firm value under any reporting strategy. This minimum value occurs if investors know with certainty that $S_t = S^l$. With these specifications, wages satisfy

$$\begin{aligned} W_t &= \alpha^*\left[V_t - V_{\min}\right] \\ &= \alpha^*\left[\mathbb{E}\left(C_t \mid \Omega_t\right) - S^l\right], \end{aligned} \tag{4.6}$$

which ensures that the minimum compensation in any period is zero. We further note that since $V_t - V_{\min}$ is a weakly positive random variable, increasing α^* drives up both the average wage and the sensitivity of wages to firm value. This feature is common to many executive compensation plans that increase pay sensitivity by granting restricted stock or stock options.

One type of equilibrium is immediately clear.

Proposition 4. *In the infinite horizon, value-based wage model, any uninformative strategy is an equilibrium strategy. In any such equilibrium the firm always has the constant value* $V_t = [S^l + (1-\alpha^*)\, p\left(S^h - S^l\right)]/(1-\beta)$ *and the manager always has wages of* $W_t = \alpha^* p\left(S^h - S^l\right)$ *per period.*

We refer to this benchmark as the *uninformative equilibrium.*

The main informative strategy of interest is the *partial revelation (PR) strategy.* When following this strategy, the manager declares for all $0 \leq \Psi_{t-1} \leq 1$ and for $i \in \{h, l\}$,

$$\begin{aligned} R^{PR}\left(N^h, S^i, \Psi_{t-1}\right) &= R^i \\ R^{PR}\left(N^l, S^i, \Psi_{t-1}\right) &= R^h. \end{aligned}$$

Thus, the manager announces his actual signal when his non-wage income is high, and declares high earnings when his non-wage income is low. This strategy is similar to the single-period strategy of the same name, but the declarations must now be made for all possible prior beliefs Ψ_{t-1}.

4.3. Incentive Compatibility Under the PR Strategy

This subsection characterizes the complete set of incentive compatibility conditions that are necessary and sufficient for existence of the PR equilibria. Since we focus on Markov perfect Bayesian equilibrium, as described in Section 2.5, incentive compatibility must hold at every point in the Markov state space $(N_t, S_t, \Psi_{t-1}) \in \left\{N^h, N^l\right\} \times \left\{S^h, S^l\right\} \times [0,1]$. Using the economic structure of the problem, we show that if PR incentive compatibility holds at three specific points in the state space, then it holds everywhere.

Denote by $J^{PR}(N_t, S_t, \Psi_{t-1})$ the manager's intertemporal expected utility in state (N_t, S_t, Ψ_{t-1}) under the assumption that he always follows the PR strategy, and that investors price securities with this knowledge. The function $J^{PR} : \left\{N^h, N^l\right\} \times \left\{S^h, S^l\right\} \times [0,1] \to \mathbb{R}^1$ satisfies the fixed point

$$J^{PR}(N_t, S_t, \Psi_{t-1}) = \mathbb{E}_{PR}\left[u(c_t) + \beta J^{PR}(N_{t+1}, S_{t+1}, \Psi_t) \,|N_t, S_t, \Psi_{t-1}\right], \qquad (4.7)$$

where $\mathbb{E}_{PR}$ denotes that expectations are conditioned on the PR strategy. Unlike the Bellman equation (2.6), the fixed point (4.7) does not require that the PR policy be chosen optimally. Instead, the function J^{PR} simply gives the value of following the PR policy without considering whether this policy is maximizing.[18]

To demonstrate an equilibrium for any choice of initial beliefs Ψ_0, it is necessary and sufficient to show that the Bellman optimality property (2.6) holds at every point (N_t, S_t, Ψ_{t-1}) in the state space.[19] Let

$$\mathbb{E}_t\left(J_{t+1}^{PR}|R_t\right) \equiv \mathbb{E}\left[J^{PR}\left(N_{t+1}, S_{t+1}, \Psi_t\right)|R_t, N_t, S_t, \Psi_{t-1}\right].$$

Also define

$$J'\left(R_t; N_t, S_t, \Psi_{t-1}\right) \equiv u\left[N_t + W^{PR}\left(R_t, \Psi_{t-1}\right)\right] + \beta\mathbb{E}_t\left(J_{t+1}^{PR}|R_t\right),$$

where W^{PR} is the value-based wage when investors price securities assuming that the manager follows the PR strategy. Using the function J', we can consider the value of continuing with the PR strategy (if $R_t = R^{PR}\left(N_t, S_t, \Psi_{t-1}\right)$) as well as the value of deviating.

Let $R^{\tilde{}PR}$ denote the strategy that makes the opposite declaration from R^{PR} in every state (N_t, S_t, Ψ_{t-1}). We can now conveniently summarize the complete set of incentive compatibility conditions. An equilibrium exists for any choice of initial beliefs Ψ_0 if and only if

$$J'\left(R^{PR}\left(N_t, S_t, \Psi_{t-1}\right); N_t, S_t, \Psi_{t-1}\right) \geq J'\left(R^{\tilde{}PR}\left(N_t, S_t, \Psi_{t-1}\right); N_t, S_t, \Psi_{t-1}\right) \tag{4.8}$$

for all $N_t \in \left\{N^h, N^l\right\}$, $S_t \in \left\{S^h, S^l\right\}$, and $\Psi_{t-1} \in [0,1]$. Incentive compatibility is thus described by four functional inequalities, each of which corresponds to a different $\{N, S\}$ couplet, and has a domain over the continuous range of prior beliefs $\Psi_{t-1} \in [0,1]$.

The economic structure of the infinite-horizon model allows us to dramatically reduce the dimensionality of the incentive compatibility conditions (4.8). Define the current utility benefit from wages of making a high forecast:

$$\Delta_u^{PR}\left(N_t, \Psi_{t-1}\right) \equiv u\left(N_t + W^{PR}\left(R^h, \Psi_{t-1}\right)\right) - u\left(N_t + W^{PR}\left(R^l, \Psi_{t-1}\right)\right). \tag{4.9}$$

Also define the future utility benefit through reputation of making a low forecast:

$$\Delta_J^{PR}\left(N_t, S_t, \Psi_{t-1}\right) \equiv \beta\left[\mathbb{E}_t\left(J_{t+1}^{PR}\middle|R^l\right) - \mathbb{E}_t\left(J_{t+1}^{PR}\middle|R^h\right)\right]. \tag{4.10}$$

[18]Specific values of the function J^{PR} are easily calculated using standard methods: (i) discretize arbitrarily finely the continuous variable $\Psi_{t-1} \in [0,1]$ from the mixed discrete-continuous state-space, and (ii) use the matrix inversion for a discrete state space described in Appendix B.

[19]If instead of focusing on the existence of equilibrium for any choice of initial beliefs Ψ_0, we take as given a specific value of $\Psi_0 \in [0,1]$, the equilibrium conditions are potentially weaker. This is explained in Appendix D.

To obtain important properties of (4.9) and (4.10), the following proposition first characterizes the underlying wage and belief updating functions, and then gives the implications for Δ_u^{PR} and Δ_J^{PR}:

Proposition 5. *(A) For* $0 \leq \Psi_{t-1} \leq 1$*, PR wages satisfy: (i)* $W^{PR}\left(R^l, \Psi_{t-1}\right) = 0$*, (ii)* $W^{PR}\left(R^h, \Psi_{t-1}\right) > 0$ *strictly increases in* Ψ_{t-1}*, and is given by the analytical formula in the Appendix.*

(B) For $0 \leq \Psi_{t-1} \leq 1$*, PR belief updating satisfies: (i)* $\Psi_t\left(R_t = R^l; C_t, \Psi_{t-1}\right) = 1$*, (ii)* $\Psi_t\left(R_t = R^h; C_t = C^l, \Psi_{t-1}\right) < \Psi_t\left(R_t = R^h; C_t = C^h, \Psi_{t-1}\right) < 1$*, where both updating functions in the inequality weakly increase in* Ψ_{t-1} *and are given by analytical formulas in the Appendix.*

(C) For all N_t, S_t, Ψ_{t-1}*, (i) the current utility benefit of a high report satisfies* $\Delta_u^{PR}\left(N_t, \Psi_{t-1}\right) \geq 0$ *and weakly increases with* Ψ_{t-1}*, (ii) The future utility benefit of a low report satisfies* $\Delta_J^{PR}\left(N_t, S_t, \Psi_{t-1}\right) \geq 0$ *and weakly decreases with* Ψ_{t-1}*.*

One of the key elements leading to Proposition 5 is that a low declaration reveals $\left(N^h, S^l\right)$. Further, the credibility of a high declaration, and hence the wage and derived utility, depend positively on prior reputation. Finally, a low cash flow results in a lower reputation than a high cash flow because the manager's signal S_t is informative and the state N_t is persistent.

Using Δ_u^{PR} and Δ_J^{PR}, we can now rewrite the incentive compatibility conditions (4.8) as

$$\Delta_u^{PR}\left(N^h, \Psi_{t-1}\right) \geq \Delta_J^{PR}\left(N^h, S^h, \Psi_{t-1}\right) \tag{4.11}$$

$$\Delta_u^{PR}\left(N^h, \Psi_{t-1}\right) \leq \Delta_J^{PR}\left(N^h, S^l, \Psi_{t-1}\right) \tag{4.12}$$

$$\Delta_u^{PR}\left(N^l, \Psi_{t-1}\right) \geq \Delta_J^{PR}\left(N^l, S^h, \Psi_{t-1}\right) \tag{4.13}$$

$$\Delta_u^{PR}\left(N^l, \Psi_{t-1}\right) \geq \Delta_J^{PR}\left(N^l, S^l, \Psi_{t-1}\right), \tag{4.14}$$

for $0 \leq \Psi_{t-1} \leq 1$. Using the fact that (4.14) implies (4.13), and also using the sign of the derivatives established in Proposition 5, we show in the following proposition that demonstrating incentive compatibility at three specific points in the state space is equivalent to satisfying (4.11-4.14).

Proposition 6. *In order to satisfy the incentive compatibility conditions* (4.8) *for the PR strategy at every point* (N_t, S_t, Ψ_{t-1}) *in the state space of the infinite horizon game, the conditions*

$$\begin{aligned} C1': &\quad \Delta_u^{PR}\left(N^h, 0\right) \geq \Delta_J^{PR}\left(N^h, S^h, 0\right) \\ C2': &\quad \Delta_u^{PR}\left(N^h, 1\right) \leq \Delta_J^{PR}\left(N^h, S^l, 1\right) \\ C3': &\quad \Delta_u^{PR}\left(N^l, 0\right) \geq \Delta_J^{PR}\left(N^l, S^l, 0\right), \end{aligned}$$

are necessary and sufficient. Equivalently, these conditions are necessary and sufficient to ensure that the infinite horizon PR strategy is an equilibrium strategy for any choice of initial beliefs $0 \leq \Psi_0 \leq 1$*.*

Thus, the economic structure of the infinite-horizon model allows us to replace the complete set of incentive compatibility constraints represented by the four functional inequalities (4.8) with an equivalent but lower-dimensional set of three point inequalities $C1' - C3'$. These conditions guarantee that no matter what the initial beliefs or how the game stochastically evolves over time, the incentive compatibility constraints of the PR equilibrium hold at every possible point in the state space and hence will always be satisfied.

4.4. Existence of PR Equilibria

We now demonstrate that the set of parameters for which the PR equilibrium exists is large, and give a constructive method of identifying PR equilibria. We first show

Lemma 1. *A necessary condition for* $C1'$ *and* $C2'$ *to hold is*

$$\Delta_J^{PR}\left(N^h, S^h, 0\right) \leq \Delta_J^{PR}\left(N^h, S^l, 1\right), \tag{4.15}$$

which requires:

$$\Psi^h\left(R_t = R^h, C_t = C^h, \Psi_{t-1} = 0\right) \geq \Psi^h\left(R_t = R^h, C_t = C^l, \Psi_{t-1} = 1\right). \tag{4.16}$$

The inequality (4.15) establishes that in equilibrium, the expected benefit of building reputation must be more influenced by the manager's signal ($S_t = S^l$ vs. $S_t = S^h$) than by his prior reputation ($\Psi_{t-1} = 0$ vs. $\Psi_{t-1} = 1$). According to (4.16), this requires that investor beliefs are greatly influenced by the observed cash flow C_t.

The following lemma establishes that the conditions required by Lemma 1 are satisfied when the manager's signal is sufficiently informative:

Lemma 2. *For* λ *close enough to one, the necessary equilibrium conditions (4.15) and (4.16) are satisfied.*

Intuitively, the informativeness of cash flows to investors depends on the accuracy of the manager's signal. For $\lambda = 1$, if investors observe $\left\{R_t = R^h, C_t = C^l\right\}$ then they are certain that $N_t = N^l$, and the right hand side of (4.16) is zero. Lemma 2 tells us that signal informativeness in the neighborhood of one guarantees (4.15).

For $\lambda < 1$, determining whether the signal is informative enough to satisfy (4.15) in general requires a complete specification of the model parameters and utility function that determine the reputation benefit Δ_J^{PR}. However, since (4.16) is a necessary condition for (4.15) to hold, and depends only on the belief updating function Ψ^h, we can eliminate excessively weak signal precision values with the following result:

Lemma 3. *Signal precision λ must satisfy*

$$\lambda \geq \lambda_{\min}(p, q),$$

where $\lambda_{\min}(p, q)$ decreases in p, increases in q, and has the closed-form expression given in the Appendix.

Figure 1 shows the minimum bounds placed on λ for $p \in \{0.05, 0.5, 0.95\}$ and $0.5 \leq q \leq 1$. The minimum λ increases in q for all three values of p. When the manager's type is more persistent, investor beliefs about type are more heavily influenced by their prior period beliefs. The signal λ must be correspondingly stronger in order for the manager's declarations to have sufficient impact to satisfy (4.16). A larger value of p also increases $\lambda_{\min}$ because when $S_t = S^l$ the manager's declaration is informative about his type, but when $S_t = S^h$ it is not. Thus, for low p, the manager's report is on average more informative holding all else constant, and a lower signal precision λ can be accommodated.

Lemmas 2 and 3 thus respectively characterize a sufficient condition $[\lambda = 1]$ and a necessary condition $[\lambda \geq \lambda_{\min}(p, q)]$ for signal informativeness to be strong enough that (4.15) holds. A highly informative signal creates a wedge between the reputation benefits conditional on $S_t = S^h$ versus $S_t = S^l$.

Using these lemmas, we now develop a theorem that ensures the inequalities $C1'$–$C3'$ can be satisfied by altering the concavity of u and the subjective discount rate β. We define

$$W_h^{\min} \equiv \min_{0 \leq \Psi_{t-1} \leq 1} \left[W^{PR}\left(R^h, \Psi_{t-1}\right) \right] = \alpha^* \left(S^h - S^l\right) \frac{p}{p + (1-p)q}$$

and denote the partial parameter vector $\Theta \equiv \left\{N^h, N^l, S^h, S^l, q, p,\ \alpha^*\right\}$. The condition

$$C\Theta: \qquad N^l + W_h^{\min} < N^h$$

guarantees a minimum level of variability in non-wage income. We then show:

Theorem (Existence of PR Equilibria). *If and only if the vector Θ satisfies condition $C\Theta$, then for λ sufficiently close to one there exists (i) a nondenumerable set $U_{\Theta,\lambda}$ of finite, concave, weakly increasing functions $u(c)$ on the interval $\left[N^l, N^h + W^{\max}\right]$, and (ii) a range of subjective discount rates, $\beta \in \left\{\left[\beta_{\Theta,u,\lambda}^{\min}, \beta_{\Theta,u,\lambda}^{\max}\right] \cap [0,1)\right\}$, where $0 < \beta_{\Theta,u,\lambda}^{\min} < 1$ and $\beta_{\Theta,u,\lambda}^{\min} \leq \beta_{\Theta,u,\lambda}^{\max} \leq 1$, for which the infinite-horizon PR equilibrium holds with utility function $u(c)$, parameters Θ, λ, β, and any initial beliefs $0 \leq \Psi_0 \leq 1$.*

The concavity of $u \in U_{\Theta,\lambda}$ and the discount factor β play a central role in proving this theorem.

The existence of PR equilibrium has four economic components. First, the manager's signal must be strong enough to create the potential that if $N_t = N^h$ the manager behaves differently when $S_t = S^h$ than when $S_t = S^l$ for any reputation level (Lemmas 1-3). Second, the variability of non-wage income must be sufficiently large relative to the minimum wage for a high declaration (Condition $C\Theta$). Third, the utility function must be sufficiently flat for large c and concave (Proof of Theorem). Fourth, given $u(.)$, the value of β must balance current wage and future reputation benefits (Proof of Theorem).

4.5. Self-Insurance Equilibrium Examples

We now give examples in which the PR equilibrium holds. Throughout the remainder of the paper we assume the manager has power utility function $u(c) = (1-\gamma)^{-1} c^{1-\gamma}$.

The following parameters provide a useful base case:

$$\begin{array}{ccccc} p = .5 & \lambda = .9 & \alpha^* = .037 & \gamma = 10 & \beta = 0.95 \\ q = 0.75 & N^l = .01 & N^h = 5 & C^l = 0 & C^h = 250. \end{array}$$

We easily verify that $\lambda \geq \lambda_{\min}(p, q)$, satisfying the necessary condition for PR equilibrium given in Lemma 3. Further, condition $C\Theta$ is satisfied since $N^l + W_h^{\min} < N^h$. We use the contraction mapping described in Appendix B to calculate the function J^{PR} defined in equation (4.7), and plot the state-contingent values of this function in Figure 2. The derived utility of the PR policy strictly increases in reputation Ψ_{t-1} for all combinations of (N_t, S_t) except (N^h, S^l). In this case, the manager's previous reputation is irrelevant because his low report tells investors that he is in state N^h with probability one. Otherwise, the upward slope of derived utility reflects the benefit of reputation.

We can confirm in two ways that, given the above choice of parameters, the PR strategy is part of a Markov perfect Bayesian equilibrium. First, as suggested by equation (4.8), we check that for all points in the state space (N_t, S_t, Ψ_{t-1}) the manager has no incentive to deviate from the PR strategy. This approach is implemented following the procedure described in Appendix B. Second, Proposition 6 establishes that the three conditions $C1' - C3'$ are equivalent to demonstrating incentive compatibility at all points in the state space. The following discussion demonstrates the implementation of both approaches, and provides a graphical depiction of the incentive compatibility conditions.

Figure 3 Panel A plots $\Delta_u^{PR}(N^h)$ and $\Delta_J^{PR}(N^h, S^j)$ for $j \in \{h, l\}$ under the base parameters. When $S_t = S^l$, the reputation benefit of declaring R^l is given by $\Delta_J^{PR}(N^h, S^l)$, and exceeds the current utility benefit $\Delta_u^{PR}(N^h)$ of declaring R^h for all values of Ψ_{t-1}. Conversely, when $S_t = S^h$, the current utility benefit of declaring

R^h exceeds the future reputation benefit of declaring R^l. The plot thus demonstrates incentive compatibility of the PR policy at all points in the state space where $N_t = N^h$. As required by Proposition 5, $\Delta_J^{PR}\left(N^h, S^j\right)$ weakly decreases and $\Delta_u^{PR}\left(N^h\right)$ weakly increases in Ψ_{t-1}. Because of these slopes, satisfying the conditions $C1'$ and $C2'$ implies incentive compatibility over the entire range of Ψ_{t-1} (Proposition 6). Although omitted from the figure for brevity, condition $C3'$ also holds. This incentive compatibility condition for $N_t = N^l$ is not close to binding, and the PR equilibrium for the base parameters is confirmed.

We now consider how small changes in parameter values affect the incentive compatibility conditions and existence of PR equilibrium. Panel B shows the impact of decreasing information precision to $\lambda = 0.89$. The functions Δ_J^{PR} increase since acquired reputation deteriorates more slowly when the manager has a weaker signal. The current utility benefit Δ_u^{PR} is almost unchanged in this example, but it is straightforward to show that wages are less sensitive to the manager's declaration when information precision is lower, hence Δ_u^{PR} must weakly decrease when λ decreases. As a result, condition $C1'$ is violated, and the PR equilibrium does not hold. By similar logic, Panel C shows that increasing information precision to $\lambda = 0.91$ results in a violation of condition $C2'$.

Panels D and E show the respective impacts of decreasing or increasing the subjective discount rate β. The discount rate has no impact on Δ_u^{PR}, and Δ_J^{PR} increases in β. In Panel D, $C2'$ is violated, while in Panel E both $C1'$ and $C2'$ continue to be satisfied.

Panel F shows the entire region of PR equilibrium existence in the space of (λ, β), holding the other base parameters constant. The labeled points show the parameter locations used in Panels A-E, where A and E are inside the equilibrium region, and the other points are outside. Equilibria exist for values of λ approximately between 0.87 and 0.905. As λ increases within this range, β increases as well to balance reputation and current wage concerns.

Figure 4 shows how the region of PR equilibrium existence changes with variations in other base parameters. Panel A is identical to Figure 3 Panel F except for scale. Each consecutive panel holds all parameters from the preceding panel constant, except for one. In Panel B, we increase the high non-wage income level to $N^h = 6.2$, which decreases current wage concerns relative to reputation. As a result, λ must rise or β must fall to compensate, and the region of equilibrium existence shifts down and to the right. Panel C decreases the non-wage income persistence to $q = 0.6$, which reduces the importance of reputation and moves the equilibrium region up and to the left. In Panel D, we reduce pay sensitivity to $\alpha = 0.034$, moving the equilibrium region back to the right. Panel E reduces risk aversion to $\gamma = 5$, diminishing the self-insurance motive, and Panel F further reduces pay sensitivity to $\alpha = .025$. The examples in this section thus confirm existence of the PR equilibrium for a broad range of parameter values.

4.6. Reputation as Self-Insurance

The PR equilibrium demonstrates a novel role for non-trivial and time-varying reputation for honesty among managers and analysts. Even though wages do not explicitly punish or reward accuracy, endogenous concern for reputation still enforces some degree of information revelation. This derives from otherwise uninsurable variation in non-wage income, which gives rise to a consumption smoothing motive: The manager builds reputation when times are good by reporting truthfully, and uses reputation in times of need by overstating forecasts.

A negative report always gives instant credibility. Investors can be sure after a low signal that the manager has high non-wage income. It is also likely that the manager will have high non-wage income in the following period, in which case he will report truthfully then as well. Low reports thus build reputation. By contrast, a string of high earnings reports gradually erodes reputation. Investors become increasingly concerned that the high reports reflect the manager's need for current income rather than the true prospects of the firm.

This environment is stationary in the sense that unconditional statistical properties are constant through time. By contrast, models with exogenous fixed types usually result in increasing certainty over time as information accumulates (e.g., Diamond, 1989). In our model, all managers sometimes lie and sometimes tell the truth. The propensity of the manager to tell the truth as estimated by the outside world, otherwise called his reputation, varies upward and downward. Reputation is thus persistent but transient.

The PR equilibrium is consistent with existing empirical facts. Reports have an upward bias, which coincides with managerial and analyst "overoptimism" that has been documented frequently in the literature. In the PR equilibrium, value-based compensation gives a direct incentive through current wages to inflate forecasts. Interestingly, if this incentive is too strong and the manager always overestimates, then investors ignore reports and the manager loses the ability to manipulate price. The PR equilibrium thus requires a balance over time between current wage concerns and reputation building. Since investors rationally anticipate the manager's reporting bias and are not misled on average, the equilibrium strategy of the manager optimizes over when to mislead and when to report accurately.

Stickel (1992) studies the price impact of recommendations by analysts with different reputations, where membership on the Institutional Investor All-America Research Team is used as a proxy for strong reputation. He finds that positive earnings reports have greater price impact when they are delivered by analysts with higher reputation, which is consistent with the reputation effects in the PR equilibrium. Stickel also finds that negative analyst reports have price impact that is unaffected by the prior reputation of the analyst. This is again consistent with the PR equilibrium, since a negative report itself delivers instant credibility and instant reputation.

Finally, the price impact of reports is positively correlated with the recent forecast accuracy of the analyst. This occurs because the unobservable state variable for non-wage income is persistent and determines the analyst's willingness to report his signal truthfully. This property of the PR equilibrium is consistent with empirical findings in Womack (1996) and Cooper, Day, and Lewis (2001).

Other models may be consistent with the empirical facts above, and hence it is useful to consider what new implications are suggested by the PR equilibrium. The key insight of our model is that consumption smoothing may be an important motivator of managerial reporting. Conditionally, we expect insiders with a high marginal utility of current income to be relatively overoptimistic. To test the theory, we would then want to identify empirical proxies for temporal variations in the insider's marginal utility. We leave the identification of such proxies for future research.

5. Welfare Comparisons

The preceding sections examined pooling, partial revelation, and full revelation equilibria, implemented by different compensation arrangements. Naturally, we would like to draw welfare comparisons across the three kinds of equilibria.

To model the gains from accurate reporting, we assume that investors plan end-of-period consumption at the beginning of each period. If dividends received differ from planned consumption, investors pay a "recontracting cost" to adjust consumption. This is a deadweight loss, and motivates why information is useful.[20]

Let $\widehat{\gamma}_t$ denote planned consumption and γ_t actual consumption. The planned consumption is determined by information available at the beginning of period t, and can be written as a function $\widehat{\gamma}_t = \widehat{\gamma}(\Omega_t)$. Recontracting costs are specified to be linear in the absolute difference between planned and actual consumption: $\xi_t = \xi\,|\widehat{\gamma}_t - \gamma_t|$. All cash flows must be consumed when they arrive, giving $C_t - W_t = \gamma_t + \xi_t$.

It is straightforward to observe that

Proposition 7. *Expected recontracting costs under FR, PR, and PO are ranked respectively by* $\mathbb{E}_{FR}(\xi_t\,|\Phi_{t-1}) \leq \mathbb{E}_{PR}(\xi_t\,|\Phi_{t-1}) \leq \mathbb{E}_{PO}(\xi_t\,|\Phi_{t-1})$*, where algebraic expressions for each quantity are given in the Appendix.*

Thus, if the sole objective is to maximize information revelation, (e.g., the point of view of a regulator), then the flat-wage or accuracy-based compensation that lead to FR

[20] In practice, an additional and likely more important reason for desiring accurate asset prices is to achieve efficient allocation of capital. We use the recontracting cost as a simpler reduced form modelling device.

are optimal. However, given the potential tradeoff between information revelation and risk-sharing in our model,[21] it is not surprising that Pareto comparisons of the three types of equilibria may not lead to a unique preference.

Previous literature (e.g., Hirshleifer, 1971) makes the general point that full revelation may not be universally desired. Hirshleifer considers a setting where an uninformed risk-averse agent has a payoff tied to the value of an asset. The agent prefers that less rather than more information be revealed to markets, since information increases the variability of his payoff.

Our setting differs since both our PO and FR equilibria are consistent with a flat wage, and thus our manager need not be harmed by stochastic wage variations under full revelation. Rather, our manager benefits under PR (relative to both PO and FR) because he can manipulate his wages to offset an outside source of variation in non-wage income. This self-insurance motive for the manager to prefer partial, but controlled, information releases differs from the general private aversion to information suggested by Hirshleifer.

The remainder of this section focuses on *interim Pareto dominance*, in which both investors and the manager are better off in every possible state of the world. A weaker requirement is *ex ante Pareto dominance*, in which both investors and the manager are better off at $t = 0$ when there is some known distribution of initial states of the economy. We make direct comparisons between each type of equilibrium, and find that every PO equilibrium is Pareto dominated, while either PR or FR can Pareto dominate the other depending on the tradeoff between information benefits and optimal risk-sharing.

5.1. Full Revelation Compared to Pooling

In comparing FR to PO, we immediately observe

Proposition 8. *For any pooling equilibrium, there exists a full revelation equilibrium that Pareto dominates it.*

The proof is direct. For any pooling equilibrium with wage sensitivity α^* and wage $W_0 = \alpha^* \mathbb{E}\left(C_t - S^l\right)$, consider the separating equilibrium with the identical flat wage. The manager is paid the same, but expected recontracting costs are lower under full revelation.

[21] If there are many investors, each facing recontracting costs, and only one manager who is concerned with consumption risk, it would seem that aggregate recontracting costs could easily dominate any risk-sharing considerations. We clarify that our measure of recontracting costs is an aggregate measure based on a percentage of the difference between planned and actual aggregate consumption. This of course necessitates that individual recontracting costs be extremely small to be comparable in aggregate to the certainly-equivalent of the manager's risk-sharing considerations.

5.2. Partial Revelation Compared to Pooling

Investors clearly have lower recontracting costs under PR compared to pooling. A second factor is that under PR the manager has a stochastic rather than a fixed wage. Investors can nonetheless pay a lower average wage under PR and still make the manager better off than under pooling. This is because PR allows the manager to manipulate his reporting strategy to offset variation in non-wage income.

We now outline an argument showing the preference of the manager for PR to PO wages. Assume the same average wage in both equilibria, i.e., $\alpha_{PR} = \alpha_{PO}$. Under the PR equilibrium, the manager could follow a strategy of always calling out R^h, which would give a strictly higher wage in each period than under the pooling equilibrium. Since the PR equilibrium strategy dominates always reporting R^h, we can conclude that the manager prefers the PR equilibrium to the PO equilibrium with the same average wage.[22]

Investors can thus pay a weakly lower average wage to the manager in PR than PO, leave the manager at least as well off in every state of the world, and still be better informed. Figure 5 confirms this reasoning. Recall our previous example in which the wage sensitivity under PR was exogenously assumed to be $\alpha_{PR} = 0.037$. Consider now a pooling equilibrium with a flat wage and $W_0 = \alpha_{PO}\mathbb{E}\left(C_t - S^l\right)$, where $\alpha_{PR} = \alpha_{PO}$. Average wages under the two equilibria are identical, but Figure 5 shows the derived utility of the manager is much higher in every state under PR than PO. We then increase α_{PO} until the manager is indifferent between the two equilibria in some state of the world, and find that this occurs at $\alpha_{PO} = 0.0438$. Investors thus pay a lower average wage under PR, but the manager strictly prefers PR in every state of the world except one, in which he is indifferent. Since recontracting costs are also lower under PR, we confirm that PR Pareto dominates PO.

5.3. Partial Revelation Compared to Full Revelation

Full information revelation clearly has an informational advantage relative to partial revelation. However, the last section showed that the manager can be better off with a lower average wage and the ability to manipulate wage under PR than with a flat wage. If this effect is strong enough then PR can dominate the full revelation equilibrium (FR) as well.

To illustrate this, we provide in Figure 6 a numerical example where PR Pareto dominates FR, using the base parameters detailed previously. We first suppose there are no recontracting costs so that $\xi = 0$. Our previous analysis has shown that with $\alpha = 0.037$ and PR, wages could be as high as $\alpha = 0.0438$ under FR and the manager would still be better off in every state of the world under PR. Since investors are paying

[22]We thank a referee for suggesting this argument.

a lower average wage and there is no difference in recontracting costs, we conclude that PR is Pareto preferred to FR when $\xi = 0$.

As we increase $\xi > 0$, expected recontracting costs rise faster under PR than FR. Eventually, this difference outweighs the lower wages under PR in some state of the world. Here we will have found the maximum value of ξ at which PR Pareto dominates FR. Figure 6 shows that PR is Pareto preferred when $\xi \leq 0.00513$. This is intuitively sensible. If recontracting costs are very high, then good information is more important and FR offers larger benefits relative to PR. In our example, when aggregate recontracting costs are less than one half of one percent of the difference between planned and actual consumption, then PR Pareto dominates FR.

Note that the interim Pareto dominance criterion still provides substantial surplus for the PR equilibrium. The manager and investors are indifferent in one state of the world, but strictly better off in every other state of the world. It is easy to conclude that the PR equilibrium is *ex ante* Pareto preferred to FR for even larger values of the recontracting cost parameter ξ.

6. Expanded Message Space, Dynamic Contracts, and the Revelation Principle

This section uses the revelation principle to obtain an equilibrium with the same risk-sharing benefits as the self-insurance (PR) equilibrium, while attaining full informational efficiency. To implement this, we must extend the analysis in two directions: 1) dynamic contracts, and 2) an expanded message space.

More concretely, previous sections considered simple static contracts where the current wage W_t is fully determined by the contemporaneous stock price V_t, report R_t, and cash flow C_t. We now allow the set of contractible variables for the time t wage to include the entire history of these variables: $\{V_s, R_s, C_s\}_{s=0}^{t}$.[23] Additionally, we allow the report to be any one of four messages: $R_t \in \{(N^h, S^h,), (N^h, S^l), (N^l, S^h), (N^l, S^l)\}$, as opposed to the restricted space of only two messages considered earlier. This expanded message space is large enough to represent the four possible states of nature at time t.

To use the revelation principle to achieve an informationally efficient version of the PR equilibrium, consider the following: In the PR equilibrium, current wage W_t is a function of the current firm value V_t. In turn, V_t is determined by the manager's

[23] As previously, we assume that a contract giving contingent wages for all $t \geq 0$ is fully specified at $t = 0$, and there is no renegotiation.

reputation Π_t and his report R_t. We can thus write

$$\begin{aligned} W_t^{PR} &= W^{PR}\left(V_t^{PR}\right) \\ &= W^{PR}\left(\Pi_t^{PR}, R^{PR}\left(\Pi_t^{PR}, N_t, S_t\right)\right), \end{aligned}$$

where R^{PR} denotes the reporting strategy of the PR equilibrium, and PR superscripts on other variables denote values under the PR equilibrium. Next, note that using the recursions (2.1) and (2.2), allows us to write reputation Π_t^{PR} as a function of the entire history of reports and cash flows $\left\{R_s^{PR}, C_s\right\}_{s=0}^{t}$, and the initial beliefs Ψ_0.

Under the revelation principle, the manager honestly reports the true state,

$$\widehat{R}_t = (N_t, S_t).$$

The contract must then specify wages that implement identical payoffs to the original PR equilibrium. To do this, the wage contract must explicitly specify the function Π_t^{PR}, as well as the R^{PR} strategy. When the declaration $\widehat{R}_t$ is made, it is converted into the PR message $R^{PR}(\widehat{R}_t)$. The value of Π_t^{PR} is then calculated using the formula specified by the contract, and the wage W_t^{PR} is paid. This mechanism guarantees that wages will be identical to the wages under the PR equilibrium, while providing full informational efficiency to the investors.

The revelation principle equilibrium makes an important point: It improves on the PR equilibrium, but at the cost of requiring a highly complex dynamic contract. In particular, the revelation principle contract must specify payoffs as a nonlinear function of past reports (which include both signal and personal wealth) and cash flow realizations.

By contrast, the PR equilibrium appears quite straightforward. Instead of writing the function Π_t^{PR} into the wage contract, reputation appears endogenously in the value function of the firm. This allows that wages can be specified simply as a linear function of the current stock price. Thus, the efficiency of the revelation principle equilibrium requires implementing a highly complex dynamic contract. On the other hand, the simplicity of the PR equilibrium derives from the rational updating of competitive investors concerned with correctly valuing the firm.

7. Conclusion

We develop a cheap-talk message game (Crawford and Sobel, 1982; Sobel, 1985) where a single informed and self-interested manager may sometimes reveal information truthfully to investors, even though his stock-based wage compensation would seem to give incentives to overstate firm value. The manager is not permitted to trade or sell his information. Instead, he uses his reporting strategy to manipulate firm value and hence his wage in a manner that offsets variation in otherwise uninsurable non-wage income.

When non-wage income is low, the manager reports high earnings; when non-wage income is high, he reports truthfully. A low report tells investors that the manager can currently afford to be honest, and that he will likely be honest in the near future. Because of this reputation for honesty, subsequent positive reports have greater price impact. Reputation thus derives from a self-insurance motive: The manager wants to be perceived as honest so that in future bad states of the world, he can credibly overstate the prospects of the firm.

The general idea that reputation may be built up when times are good so that it can be exploited in times of need seems plausible because many corporate executives have a considerable portion of their wealth tied to the firms they manage, and consumption smoothing may be a legitimate concern. The basic idea may also have implications beyond the simple cheap-talk message game studied here. In particular, it would be interesting to extend this type of concern for reputation to a manager making real investment decisions.[24]

[24] Another idea suggested by a referee is that stochastic and persistent changes in the subjective discount rate could play the same role as non-wage income in altering marginal utilities of the manager across states. This idea is attractive because it is unlikely that such changes in preferences would be hedgeable in financial markets, consistent with our self-insurance motivation.

Appendix

A. Belief Updating

This section discusses successively belief updating in the single-period and infinite-horizon models.

A.1. Belief Updating in the Single-Period Model

When the earnings report is announced, denote investors' new beliefs about type $i \in \{h, l\}$ by $\Pi^i(R) \equiv \mathbb{P}\left(N^i \middle| R\right)$. We can compute these by calculating the values of $\Pi^{ij}(R) \equiv \mathbb{P}\left(N^i, S^j \middle| R\right)$ and noting that $\Pi^i(R) = \sum_{j\in\{h,l\}} \Pi^{ij}(R)$.

To calculate Π^{ij}, denote $\rho_k^{ij} \equiv \mathbb{P}\left(R = R^k \middle| N^i, S^j\right)$, which defines the strategy the manager is playing. For pure strategies, ρ_k^{ij} take only values of zero and one, and for mixed strategies it can be anything in-between. We can then use Bayes' rule to calculate

$$\Pi^{ij}\left(R = R^k\right) = \frac{\rho_k^{ij} p^j \Psi_0^i}{\sum_{i,j\in\{h,l\}} \rho_k^{ij} p^j \Psi_0^j},$$

where $p^h \equiv p$ and $p^l \equiv 1 - p$.

To derive the value of the firm in equation (4.2), we then use

$$\mathbb{E}\left(S \middle| R^k\right) = \sum_{i,j\in\{h,l\}} \Pi^{ij}\left(R^k\right) S^j.$$

These calculations provide analytical expressions for the updated beliefs Π and the value of the firm V after the earnings report has been declared.

When the final cash flow C is observed, beliefs must be updated again to solve for the terminal beliefs $\Psi_1^i = \mathbb{P}\left(N^i \middle| R, C\right)$, $i \in \{h, l\}$. These satisfy $\mathbb{P}\left(N^i, C \middle| R\right) / \mathbb{P}\left(C \middle| R\right)$, and summing over the two possible values of S_t gives

$$\Psi_1^i = \frac{\sum_{j\in\{h,l\}} \mathbb{P}\left(N^i, S^j, C \middle| R\right)}{\mathbb{P}\left(C \middle| R\right)}.$$

The summands in the numerator simplify[25] to $\mathbb{P}\left(N^i, S^j, C \middle| R\right) = \Pi^{ij}(R)\,\mathbb{P}\left(C \middle| S^j\right)$. Using the fact that Ψ_t^h and Ψ_t^l sum to one, we obtain for $k \in \{h, l\}$,

$$\Psi_1^i\left(C = C^k\right) = \frac{\sum_{j\in\{h,l\}} \Pi^{ij}(R)\,\lambda^{jk}}{\sum_{m,j\in\{h,l\}} \Pi^{mj}(R)\,\lambda^{jk}}. \tag{A.1}$$

For all practical purposes, off-equilibrium beliefs play no role in our model. As long as both reports are used with some positive probability in some state of the world,

[25] We use the fact that $\mathbb{P}\left(N^i, S^j, C \middle| R\right) = \mathbb{P}\left(N^i, S^j \middle| R\right) \mathbb{P}\left(C \middle| R, N^i, S^j\right)$.

the conditional beliefs above are always well-defined. Thus, the only strategy where off-equilibrium beliefs matter is when the manager declares R^h with probability one in every state.[26] This is only one of many possible pooling strategies, (including random mixing), so whether this forms an equilibrium is not of any special interest. Thus, off-equilibrium beliefs matter only for one pooling strategy that is of no particular importance.

A.2. Belief Updating in the Infinite Horizon Model

After the earnings forecast is made, beliefs satisfy $\Pi_t^i(R_t, \Psi_{t-1}) = \sum_{j\in\{h,l\}} \Pi_t^{ij}(R_t, \Psi_{t-1})$, where $\Pi_t^{ij}(R_t, \Psi_{t-1}) \equiv \mathbb{P}\left(N_t = N^i, S_t = S^j \middle| \Omega_t\right)$. Using Bayes' rule,

$$\Pi_t^{ij}\left(R_t = R^k\right) = \frac{\rho_k^{ij}(\Psi_{t-1})\, p^j \left(\sum_{m\in\{h,l\}} \Psi_{t-1}^m q^{mi}\right)}{\mathbb{P}\left(R_t = R^k \middle| \Phi_t\right)}. \tag{A.2}$$

This allows us to calculate the value V_t of the firm using (2.5).

This post-forecast updating equation $(A.2)$ slightly differs from the single-period model since the value of N_t may change over time. In particular, this necessitates summing over both possible states of N_{t-1} and using the switching probabilities q^{mi}.

After cash flows are observed, the updated beliefs are

$$\Psi_t^i\left(C_t = C^k\right) = \frac{\sum_{j\in\{h,l\}} \Pi_t^{ij}(R_t)\, \lambda^{jk}}{\mathbb{P}\left(C_t = C^k \middle| \Omega_t\right)}. \tag{A.3}$$

This expression parallels the updating equation for post-cash-flow beliefs in the single period model.

As in the single-period model, off-equilibrium beliefs play no important role. Provided that for every $\Psi_{t-1} \in [0,1]$, each report is used with positive probability in some state of the world, then the conditional beliefs above are always well-defined. Thus, the only strategies where off-equilibrium beliefs matter have some value of Ψ_{t-1} for which the manager declares R^h with probability one in every (N, S) state (pooling). Again, since the manager can instead pool by mixing in a way unrelated to the (N, S) state (babbling), an alternative strategy always exists that is similar in all respects, except that conditional probabilities are well-defined.

[26] If investors attribute any declaration of R^l to a random error unrelated to the true state, then always declaring R^h is an equilibrium strategy. This strategy also forms an equilibrium if investors believe, as in Ottaviani and Squintani (2005), that with some small probability the manager will declare truthfully, where truth is defined by the "natural meaning" of the message. For all other strategies in our model, investor beliefs are well-defined for every action and thus the specification of off-equilibrium beliefs is not important.

B. Numerical Calculation of the Value Function

Denote by ψ the column vector $(0, .01, ..., 1)'$ which discretizes the possible values of Ψ_t. We calculate the value of an arbitrary pure strategy $R \in \mathcal{R}$ at all points in the discretized state space $(N, S, \Psi_{t-1}) \in \{0,1\} \times \{0,1\} \times \psi = x$. Enumerate the possible (N, S, Ψ_{t-1}) states by $x_i \in x$, $i = 1, ..., n$, where $n = 2^2 * 101$.

The policy R on the discretized space x is an n element column vector of zeros and ones, where a zero denotes declaring R^l and a one denotes declaring R^h. Calculate for each state updated beliefs $\pi_i = \Pi(\psi_i, R(x_i))$, which gives for each state firm value v_i through (2.5) and wages w_i through (4.5). Let N_i denote the value of N in state x_i. Also denote the manager's current period utility reward in state x_i by $\nu_i = u(N_i + \alpha w_i)$, and stack in a column vector $\nu \equiv (\nu_1, ..., \nu_n)'$.

Next calculate transitions over the state space x. Use (2.2) to obtain for $j \in \{h, l\}$ the values $\Psi\left(\pi_i, C^j\right)$ and denote these $\psi'_{i,j}$. For each state $x_i \in x$, there are two possible values of future reputation. The transitions of S and N are respectively iid and first-order Markov. Combining the transitions of ψ, S, and N gives an $n \times n$ state transition matrix Σ with rows that sum to one.

We can now use standard techniques to calculate the value function. Assuming that the policy R is followed perpetually gives $J = \nu + \beta\Sigma J$, where J is an n element column vector with element i giving the manager's indirect utility in state x_i. Letting I be an $n \times n$ identity matrix, the indirect utilities are given by $J = (I - \beta\Sigma)^{-1} \nu$.

To check for equilibrium, we test whether the manager will in fact want to use the strategy $R(x_i)$ in each state x_i. For each state x_i calculate $\pi_i^- = \Pi(\psi_i, [1 - R(x_i)])$ which gives the conditional beliefs after the earnings announcement if an announcement that deviates from the strategy R is made. It is then straightforward to calculate the current utility and discounted expected indirect utility of the deviant strategy and compare their sum to J_i. The strategy $R(.)$ must be optimal for all x_i for a Markov perfect Bayesian equilibrium to exist.

C. Proofs of Propositions, Lemmas, and Theorem

C.1. Proof of Proposition 1

With flat wage compensation, the manager is indifferent across all strategies, and any strategy is an equilibrium strategy. Accuracy-based compensation induces truth-telling, and full revelation is the only equilibrium.

C.2. Proof of Proposition 2

Since any two strategies with exactly opposite reports in each state are observationally equivalent, we can without loss of generality restrict our analysis to strategies with $R\left(N^l, S^h\right) = R^h$. There are thus $2^{(4-1)} = 8$ pure strategies to consider. To index the strategies, define for any pure strategy R^i an indicator variable $\mathfrak{h}_{j,k}(i)$ that equals one if $R^i\left(N^j, S^k\right) = R^h$ and negative one otherwise. We now require that each pure strategy R^i, $i = 1,..,8$, satisfies $i = 2\mathfrak{h}_{h,h}(i) + \mathfrak{h}_{h,l}(i) + \mathfrak{h}_{l,l}(i)/2 + 4.5$ so that the index i uniquely identifies each strategy. As an example, consider the strategy R^1 for which $4\mathfrak{h}_{h,h}(1) + 2\mathfrak{h}_{h,l}(1) + \mathfrak{h}_{l,l}(1) = -7$. For this equality to hold, we must have $\mathfrak{h}_{h,h}(1) = \mathfrak{h}_{h,l}(1) = \mathfrak{h}_{l,l}(1) = -1$, so the strategy R^1 denotes declaring R^l in every state except $\left(N^l, S^h\right)$.

Now consider any single pure strategy $R^i \in \mathcal{R}$, and denote by R^{i-} the strategy which for all $j,k \in \{h,l\}$ satisfies $R^{i-}\left(N^j, S^k\right) \neq R^i\left(N^j, S^k\right)$. The strategy R^{i-} thus delivers opposite reports to R^i in every state. Also denote by $W^i(R)$ and $\Psi^i(R,C)$ the wage and reputation updating functions for investors who update using Bayes' rule assuming that the manager uses strategy R^i. For $j,k,m \in \{h,l\}$, let the functions

$$\overline{U}^i_{j,k}(R^m) \equiv U\left(N^j, W^i(R^m), \mathbb{E}\left[\Psi_1^{i,h}(R^m, C)\middle| S^k\right]\right)$$

give the manager's expected total utility in each $\{N,S\}$ state if the earnings report R^m is given.

In order for R^i to be an equilibrium strategy, it must satisfy the optimality equation (4.4). Thus, equilibrium requires that for all four combinations of $j,k \in \{h,l\}$, the inequality

$$\overline{U}^i_{j,k}\left(R^i\left(N^j, S^k\right)\right) \geq \overline{U}^i_{j,k}\left(R^{i-}\left(N^j, S^k\right)\right) \tag{C.1}$$

is satisfied.

The inequalities given by $(C.1)$ can be conveniently written using the sum of two differences. Let

$$\begin{aligned}
\Delta^i_u\left(N^j\right) &\equiv u\left(N^j + W^i\left(R^h\right)\right) - u\left(N^j + W^i\left(R^l\right)\right), \\
\Delta^i_\Psi\left(S^k\right) &\equiv \theta\mathbb{E}\left[\Psi_1^{i,h}\left(R^l, C\right) - \Psi_1^{i,h}\left(R^h, C\right)\middle| S^k\right].
\end{aligned}$$

The necessary and sufficient conditions for for R^i to be an equilibrium can now be written

$$\begin{aligned}
I^i_1: &\quad \mathfrak{h}_{h,h}(i)\left[\Delta^i_u\left(N^h\right) - \Delta^i_\Psi\left(S^h\right)\right] \geq 0 \\
I^i_2: &\quad \mathfrak{h}_{h,l}(i)\left[\Delta^i_u\left(N^h\right) - \Delta^i_\Psi\left(S^l\right)\right] \geq 0 \\
I^i_3: &\quad \mathfrak{h}_{l,h}(i)\left[\Delta^i_u\left(N^l\right) - \Delta^i_\Psi\left(S^h\right)\right] \geq 0 \\
I^i_4: &\quad \mathfrak{h}_{l,l}(i)\left[\Delta^i_u\left(N^l\right) - \Delta^i_\Psi\left(S^l\right)\right] \geq 0.
\end{aligned}$$

The remaining subsections conclude the proof.

Conclusion to Part i) By the definition of an uninformative strategy, no information is conveyed about firm value so $W^*(R^h) = W^*\left(R^l\right)$ for any uninformative strategy R^*. Also since no information is conveyed about type, $\Psi_1^{*h}\left(R^h, C\right) = \Psi_1^{*h}\left(R^l, C\right)$ for $C \in \left\{C^h, C^l\right\}$. It is therefore trivial that $I_1^*, .., I_4^*$ are satisfied with strict equality. Thus, if investors update beliefs assuming that the earnings report contains no information, the actual earnings report that is declared has no effect on the manager's utility, and any strategy is (weakly) optimal. The uninformative pure strategy R^8 as well as any uninformative mixed strategies are therefore equilibrium strategies.

Conclusion to Part ii) It is straightforward to observe that conditions $C1, .., C3$ referred to in the theorem are respectively inequalities I_1^{PR}, I_2^{PR}, and I_4^{PR} where $PR = 6$ gives the index number of the partial revelation strategy R^{PR}. We therefore need only show that I_3^{PR} is implied by $\left\{I_1^{PR}, I_2^{PR}, I_4^{PR}\right\}$. In fact, I_3^{PR} is implied by either of I_1^{PR} or I_4^{PR}.

Assume I_1^{PR} holds: Since $\Delta_u^{PR}\left(N^h\right) \leq \Delta_u^{PR}\left(N^l\right)$ by the concavity of $u\left(.\right)$, then I_3^{PR} must also hold.

Assume I_4^{PR} holds: Observe that for $k \in \{h, l\}$,

$$\Delta_\Psi^{PR}\left(S^k\right) = \theta\left\{1 - \lambda^{kh}\Psi_1^{PR,h}\left(R^h, C^h\right) + \lambda^{kl}\Psi_1^{PR,h}\left(R^h, C^l\right)\right\}.$$

Since $\lambda > .5$, then $\Psi_1^{PR,h}\left(R^h, C^h\right) > \Psi_1^{PR,h}\left(R^h, C^l\right)$, and therefore $\Delta_\Psi^{PR}\left(S^h\right) < \Delta_\Psi^{PR}\left(S^l\right)$. It is immediate that I_3^{PR} must also hold.

Conclusion to Part iii) We have analyzed equilibrium for the uninformative strategy R^8 and for the partial truth-telling strategy R^6. We must now rule out equilibria for the remaining pure strategies.

R^1 : Some algebra shows that $\Psi_1^1\left(R^l, C^h\right) > \Psi_1^1\left(R^l, C^l\right)$. We therefore have $\Delta_\Psi^1\left(S^h\right) > \Delta_\Psi^1\left(S^l\right)$, which contradicts the manager declaring R^h in $\left(N^l, S^h\right)$ and declaring R^l in $\left(N^l, S^l\right)$.⊗

R^2 : This strategy is perfectly informative about nonwage income but uninformative about the signal. It is straightforward to show $\Delta_u^2\left(N^h\right) = \Delta_u^2\left(N^l\right) = 0$ and $\Delta_\Psi^2\left(S^h\right) = \Delta_\Psi^2\left(S^l\right) = \theta$. The manager would therefore want to declare R^l in every state.⊗

R^3 : Combining I_1^3 and I_2^3 implies $\Delta_\Psi^3\left(S^h\right) \geq \Delta_\Psi^3\left(S^l\right)$. Combining I_3^3 and I_4^3 implies $\Delta_\Psi^3\left(S^h\right) \leq \Delta_\Psi^3\left(S^l\right)$. Combining these results and all of the four inequalities gives $\Delta_\Psi^3\left(S^h\right) = \Delta_\Psi^3\left(S^l\right) = \Delta_u^3\left(N^l\right) = \Delta_u^3\left(N^h\right)$. From the last equality and the concavity of $u\left(.\right)$ we can infer $\Delta_\Psi^3\left(S^i\right) = \Delta_u^3\left(N^i\right) = 0$ for $i \in \{h, l\}$. This implies that $W^3\left(R^h\right) = W^3\left(R^l\right)$, which can only hold if $\Psi_0^h = .5$. On the other hand, $\Delta_\Psi^3\left(S^i\right) = 0$ and $\theta \neq 0$ imply that $\Psi_1^3\left(R^h, C^j\right) = \Psi_1^3\left(R^l, C^j\right)$ for $j \in \{h, l\}$. Either of these equalities is inconsistent with $\Psi_0^h = .5$ and $\lambda > 0.5$.⊗

R^4 : This strategy gives $W^4\left(R^l\right) > W^4\left(R^h\right)$ and hence $\Delta_u^4\left(N^l\right) < \Delta_u^4\left(N^h\right) < 0$. Combining I_1^4 and I_3^4 gives $\Delta_u^4\left(N^l\right) \geq \Delta_u^4\left(N^h\right)$, a contradiction.$\otimes$

R^5 : This strategy is perfectly informative about the signal but uninformative about nonwage income. It gives for $j,k \in \{h,l\}$ that $\Delta_u^5\left(N^j\right) > 0$ and $\Delta_\Psi^5\left(S^k\right) = 0$. The manager would thus always want to declare R^h.$\otimes$

R^7 : This strategy gives $W^7\left(R^l\right) < W^7\left(R^h\right)$ and hence $\Delta_u^7\left(N^l\right) > \Delta_u^7\left(N^h\right) > 0$. Combining I_2^7 and I_4^7 gives $\Delta_u^7\left(N^l\right) \leq \Delta_u^7\left(N^h\right)$, a contradiction.$\otimes$

We also rule out informative three and four message pure strategies. There is one four message (fully revealing) strategy, and there are six three message strategies. All of these except one can be ruled out on the simple grounds that they have the manager fully revealing either in state $\left(N^h, S^h\right)$ or in state $\left(N^l, S^l\right)$. It is straightforward to observe that full revelation in either of these states is inconsistent with equilibrium: If the manager separates in $\left(N^l, S^l\right)$ then he guarantees the worst possible wage and reputation, and would always prefer any other declaration in this state. If the manager separates in $\left(N^h, S^h\right)$ then the corresponding message would give the highest possible wage and reputation, and in every other state he would want to deviate to send this message.

The only three or four message strategy not ruled out by this argument has the manager pooling across $\left(N^h, S^h\right)$ and $\left(N^l, S^l\right)$ and separating in the other two states. To rule this out, assume the manager sends message a in $\left(N^l, S^h\right)$, c in $\left(N^h, S^l\right)$, and b in the pooling states. We can then show that incentive compatibility for choosing a over b in state $\left(N^l, S^h\right)$ contradicts choosing b over a in state $\left(N^l, S^l\right)$. In particular, note that respectively these give

$$\begin{aligned} u\left(N^l + W(R=a)\right) - u\left(N^l + W(R=b)\right) &\geq \theta \mathbb{E}\left[\Psi_1^h\left(R=b, C \middle| S^h\right)\right] \\ u\left(N^l + W(R=a)\right) - u\left(N^l + W(R=b)\right) &\leq \theta \mathbb{E}\left[\Psi_1^h\left(R=b, C \middle| S^l\right)\right]. \end{aligned}$$

These imply

$$\mathbb{E}\left[\Psi_1^h\left(R=b, C \middle| S^l\right)\right] \geq \mathbb{E}\left[\Psi_1^h\left(R=b, C \middle| S^h\right)\right].$$

This is contradicted by the following observation: Since for $i \in \{h,l\}$, the cash flow C^i is more likely when the signal S^i is observed, expected future reputation from pooling with b must be larger when S^h is declared when S^l is declared. This contradiction rules this strategy out as an equilibrium.

C.3. Proof of Proposition 3

Some algebra gives $W_t = \frac{\alpha^*}{(1-\alpha^*)} V_t + \frac{1}{1-\alpha^*}\left[\frac{1-\beta+\alpha^*\beta}{1-\beta} c^* + (\alpha^*)^2 \mathbb{E}_t\left(\sum_{s=1}^{\infty} \beta^s V_{t+s}^*\right)\right]$. Using $\mathbb{E}_t V_{t+s}^* = \frac{1}{1-\beta}\mathbb{E}C$ for $s \geq 1$ then gives the result with coefficients $\alpha = \frac{\alpha^*}{(1-\alpha^*)}$ and $c = \frac{1}{1-\alpha^*}\left[\frac{1-\beta+\alpha^*\beta}{1-\beta} c^* + \left(\frac{\alpha^*}{1-\beta}\right)^2 \mathbb{E}C\right]$.

C.4. Proof of Proposition 4

By definition $\mathbb{E}\left(C_t \left|\Psi_{t-1}, R^h\right.\right) = \mathbb{E}\left(C_t \left|\Psi_{t-1}, R^l\right.\right) = \mathbb{E}\left(C_t\right)$ when the strategy is uninformative. The equations in the proposition then follow immediately.

C.5. Proof of Proposition 5

Following equation (4.6), wages satisfy

$$\begin{aligned} W_t &= \alpha^* \left[\mathbb{E}\left(C_t \middle| \Omega_t\right) - S^l\right] \\ &= \alpha^* \left(S^h - S^l\right) \mathbb{P}\left(S_t = S^h \middle| \Omega_t\right) \\ &= \alpha^* \left(S^h - S^l\right) \mathbb{P}\left(S_t = S^h \middle| R_t, \Psi_{t-1}\right). \end{aligned}$$

Under the PR strategy, a low declaration immediately informs investors that $S_t = S^l$, while a high declaration means either (i) $S_t = S^h$ or (ii) $S_t = S^l$ and $N_t = N^l$. Thus,

$$\begin{aligned} \mathbb{P}\left(S_t = S^h \middle| R_t = R^l, \Psi_{t-1}\right) &= 0 \\ \mathbb{P}\left(S_t = S^h \middle| R_t = R^h, \Psi_{t-1}\right) &= \frac{p}{p + (1-p)\left[\Psi_{t-1}(1-q) + (1-\Psi_{t-1})\,q\right]} \\ &= \frac{p}{p + (1-p)\left[q + (1-2q)\,\Psi_{t-1}\right]}. \end{aligned}$$

PR wages therefore satisfy

$$\begin{aligned} W^{PR}\left(R^l, \Psi_{t-1}\right) &= 0 \\ W^{PR}\left(R^h, \Psi_{t-1}\right) &= \alpha^* \left(S^h - S^l\right) \frac{p}{p + (1-p)\left[q + (1-2q)\,\Psi_{t-1}\right]}. \end{aligned}$$

It is straightforward to verify that for high declarations wages increase in prior reputation. Algebraically, we check that the derivative of $W^{PR}\left(R^h, \Psi_{t-1}\right)$ with respect to Ψ_{t-1} must be positive since it has the opposite sign of $1 - 2q < 0$. We conclude that wages increase in prior reputation.

Following the definition (4.9), we substitute $W^{PR}\left(R^l, \Psi_{t-1}\right) = 0$ and obtain

$$\Delta_u^{PR}\left(N_t, \Psi_{t-1}\right) = u\left(N_t + W^{PR}\left(R^h, \Psi_{t-1}\right)\right) - u\left(N_t\right).$$

Since $u(.)$ is increasing, we conclude that $\Delta_u^{PR}\left(N_t, \Psi_{t-1}\right)$ is weakly positive and increasing in Ψ_{t-1}, satisfying parts (i) and (iii) of the proposition.

To demonstrate (ii) and (iv), we follow (A.3) and observe that when $R_t = R^l$ the end-of-period beliefs satisfy

$$\Psi_t\left(R_t = R^l; C_t, \Psi_{t-1}\right) = 1$$

for either value of the cash flow realization C_t and any prior beliefs Ψ_{t-1}. When $R_t = R^h$, we calculate

$$\Psi_t\left(R_t = R^h; C_t = C^l, \Psi_{t-1}\right) = \frac{\mathbb{P}_t^h p(1-\lambda)}{\mathbb{P}_t^h p(1-\lambda) + \mathbb{P}_t^l\left[p(1-\lambda) + (1-p)\lambda\right]} \quad \text{(C.2)}$$

$$\Psi_t\left(R_t = R^h; C_t = C^h, \Psi_{t-1}\right) = \frac{\mathbb{P}_t^h p\lambda}{\mathbb{P}_t^h p\lambda + \mathbb{P}_t^l\left[p\lambda + (1-p)(1-\lambda)\right]} \quad \text{(C.3)}$$

where for $i \in \{h, l\}$,

$$\begin{aligned} \mathbb{P}_t^i &\equiv \mathbb{P}\left(N_t = N^h \middle| \Phi_{t-1}\right) \\ &= \mathbb{P}\left(N_t = N^h \middle| \Psi_{t-1}\right) \end{aligned}$$

are the beginning-of-period beliefs, which satisfy

$$\begin{aligned} \mathbb{P}_t^h &= \left[\Psi_{t-1} q + (1-\Psi_{t-1})(1-q)\right] \\ \mathbb{P}_t^l &= \left[\Psi_{t-1}(1-q) + (1-\Psi_{t-1}) q\right]. \end{aligned}$$

Since $q > 0.5$, the probability $\mathbb{P}_t^h$ increases in Ψ_{t-1} and $\mathbb{P}_t^l$ decreases in Ψ_{t-1}, which establishes that $\Psi_t\left(R_t = R^h; C_t, \Psi_{t-1}\right)$ increases in Ψ_{t-1} for either value of the cash flow realization.

Following the definition (4.10), we can calculate

$$\Delta_J^{PR}(N_t, S_t, \Psi_{t-1}) = \beta\left\{\begin{array}{c} \mathbb{E}\left[J^{PR}(N_{t+1}, S_{t+1}, \Psi_t = 1) \,|N_t\right] \\ -\mathbb{E}\left[J^{PR}(N_{t+1}, S_{t+1}, \Psi_t) \,\middle|R_t = R^h, N_t, S_t, \Psi_{t-1}\right] \end{array}\right\}.$$

The intertemporal utility $J^{PR}(N_{t+1}, S_{t+1}, \Psi_t)$ increases in Ψ_t, and hence the utility difference $\Delta_J^{PR}(N_t, S_t, \Psi_{t-1})$ is positive and decreases in Ψ_{t-1} for all values of (N_t, S_t), satisfying parts (ii) and (iv) of the proposition.

C.6. Proof of Proposition 6

We show that the conditions $C1'$-$C3'$ are necessary and sufficient for the complete set of incentive compatibility conditions (4.11)-(4.14) to hold. We first establish that (4.14) implies (4.13) by demonstrating

$$\Delta_J^{PR}\left(N^l, S^l, \Psi_{t-1}\right) \geq \Delta_J^{PR}\left(N^l, S^h, \Psi_{t-1}\right). \quad \text{(C.4)}$$

The quantities on both sides of (C.4) are equal to utility differences between declaring R^l and declaring R^h, in states $\left(N^l, S^l, \Psi_{t-1}\right)$ and $\left(N^l, S^h, \Psi_{t-1}\right)$ respectively. The utilities J^{PR} in states $S_t = S^l$ on the left hand side and $S_t = S^h$ on the right hand side will differ only if the manager declares R^h. This is because S_t impacts J only through investor beliefs by altering the probabilites of the C_t realizations, but when $R_t = R^l$ the manager finishes period t with reputation $\Psi_t = 1$ regardless of the C_t outcome. Algebraically, we can rewrite the utility differences for $i \in \{h, l\}$ as

$$\begin{aligned} \Delta_J^{PR}\left(N^l, S^i, \Psi_{t-1}\right) &= \mathbb{E}\left[J^{PR}\left(N_{t+1}, S_{t+1}, \Psi_t = 1\right) \middle| N_t = N^l\right] \\ &\quad - \mathbb{E}\left[J^{PR}\left(N_{t+1}, S_{t+1}, \Psi_t\right) \middle| R_t = R^h, N_t = N^l, S_t = S^i, \Psi_{t-1}\right] \end{aligned}$$

implying that (C.4) simplifies to:

$$\begin{aligned} & \mathbb{E}\left[J^{PR}\left(N_{t+1}, S_{t+1}, \Psi_t\right) \middle| R_t = R^h, N_t = N^l, S_t = S^h, \Psi_{t-1}\right] \\ \geq\ & \mathbb{E}\left[J^{PR}\left(N_{t+1}, S_{t+1}, \Psi_t\right) \middle| R_t = R^h, N_t = N^l, S_t = S^l, \Psi_{t-1}\right]. \end{aligned} \tag{C.5}$$

We established in Proposition 5 that J^{PR} increases in Ψ_t. It is also straightforward to verify from (C.2) and (C.3) that beliefs are higher following $C_t = C^h$ than if $C_t = C^l$:

$$\frac{\Psi_t\left(R_t = R^h; C_t = C^h, \Psi_{t-1}\right)}{\Psi_t\left(R_t = R^h; C_t = C^l, \Psi_{t-1}\right)} = \frac{\lambda\left(1-\lambda\right)p + \lambda^2\left(1-p\right)\mathbb{P}_t^l\left(\Psi_{t-1}\right)}{\lambda\left(1-\lambda\right)p + \left(1-\lambda\right)^2\left(1-p\right)\mathbb{P}_t^l\left(\Psi_{t-1}\right)} > 1.$$

Since the probability of drawing $C_t = C^h$ is larger for $S_t = S^h$ than for $S_t = S^l$, we conclude that (C.5) holds. This establishes that (4.14) implies (4.13).

We now want to show that (4.11) holds for all $0 \leq \Psi_{t-1} \leq 1$ if and only if it holds at $\Psi_{t-1} = 0$. This is straightforward since Proposition 6 establishes that Δ_u^{PR} is upward sloping in Ψ_{t-1} and Δ_J^{PR} is downward sloping in Ψ_{t-1}. Their difference hence increases with Ψ_{t-1}, and $C1'$ is necessary and sufficient for (4.11). Similar arguments show that $C2'$ is necessary and sufficient for (4.12) and $C3'$ is necessary and sufficient for (4.14).

C.7. Proof of Lemma 1

Combining the conditions $C1'$ and $C2'$ of Proposition 6 with the upward slope of Δ_u^{PR} given by Proposition 5, the following set of inequalities must hold in any equilibrium:

$$\Delta_J^{PR}\left(N^h, S^h, 0\right) \leq \Delta_u^{PR}\left(N^h, 0\right) \leq \Delta_u^{PR}\left(N^h, 1\right) \leq \Delta_J^{PR}\left(N^h, S^l, 1\right). \tag{C.6}$$

The relation between the endpoints establishes (4.15).

We now establish that (4.16) is a necessary condition for (4.15). An implication of (4.15) is

$$\begin{aligned} & \mathbb{E}\left[J^{PR}\left(N_{t+1}, S_{t+1}, \Psi_t\right)\middle| R_t = R^l, N_t = N^h, S_t = S^h, \Psi_{t-1} = 0\right] \\ & -\mathbb{E}\left[J^{PR}\left(N_{t+1}, S_{t+1}, \Psi_t\right)\middle| R_t = R^h, N_t = N^h, S_t = S^h, \Psi_{t-1} = 0\right] \\ \leq\ & \mathbb{E}\left[J^{PR}\left(N_{t+1}, S_{t+1}, \Psi_t\right)\middle| R_t = R^l, N_t = N^h, S_t = S^l, \Psi_{t-1} = 1\right] \\ & -\mathbb{E}\left[J^{PR}\left(N_{t+1}, S_{t+1}, \Psi_t\right)\middle| R_t = R^h, N_t = N^h, S_t = S^l, \Psi_{t-1} = 1\right] \end{aligned}$$

Since declaring $R_t = R^l$ results in a reputation $\Psi_t = 1$ regardless of the prior reputation Ψ_{t-1} or subsequent cash flow C_t, the first terms on each side of the inequality have the same value. The previous inequality thus simplifies to

$$\begin{aligned} & \mathbb{E}\left[J^{PR}\left(N_{t+1}, S_{t+1}, \Psi_t\left(R_t = R^h, C_t, \Psi_{t-1} = 0\right)\right) \middle| N_t = N^h, S_t = S^h \right] \\ \geq \; & \mathbb{E}\left[J^{PR}\left(N_{t+1}, S_{t+1}, \Psi_t\left(R_t = R^h, C_t, \Psi_{t-1} = 1\right)\right) \middle| N_t = N^h, S_t = S^l \right]. \end{aligned}$$

We decompose the expectations into the possible cash flow outcomes:

$$\begin{aligned} & \lambda \mathbb{E}\left[J^{PR}\left(N_{t+1}, S_{t+1}, \Psi_t\left(R_t = R^h, C_t = C^h, \Psi_{t-1} = 0\right)\right) \middle| N_t = N^h \right] \\ & + (1-\lambda)\,\mathbb{E}\left[J^{PR}\left(N_{t+1}, S_{t+1}, \Psi_t\left(R_t = R^h, C_t = C^l, \Psi_{t-1} = 0\right)\right) \middle| N_t = N^h \right] \\ \geq \; & \lambda \mathbb{E}\left[J^{PR}\left(N_{t+1}, S_{t+1}, \Psi_t\left(R_t = R^h, C_t = C^l, \Psi_{t-1} = 1\right)\right) \middle| N_t = N^h \right] \\ & + (1-\lambda)\,\mathbb{E}\left[J^{PR}\left(N_{t+1}, S_{t+1}, \Psi_t\left(R_t = R^h, C_t = C^h, \Psi_{t-1} = 1\right)\right) \middle| N_t = N^h \right]. \end{aligned} \tag{C.7}$$

We know that

$$\Psi_t\left(R_t = R^h, C_t = C^l, \Psi_{t-1} = 0\right) < \Psi_t\left(R_t = R^h, C_t = C^h, \Psi_{t-1} = 1\right)$$

always holds. Thus, a necessary condition for $(C.7)$ is that

$$\Psi_t\left(R_t = R^h, C_t = C^h, \Psi_{t-1} = 0\right) \geq \Psi_t\left(R_t = R^h, C_t = C^l, \Psi_{t-1} = 1\right), \tag{C.8}$$

which establishes the lemma.

C.8. Proof of Lemma 2

When $\lambda = 1$, $(C.7)$ holds with a strict inequality. To see this, observe that $(C.7)$ reduces to

$$\begin{aligned} & \mathbb{E}\left[J^{PR}\left(N_{t+1}, S_{t+1}, \Psi_t\left(R_t = R^h, C_t = C^h, \Psi_{t-1} = 0\right)\right) \middle| N_t = N^h \right] \\ \geq \; & \mathbb{E}\left[J^{PR}\left(N_{t+1}, S_{t+1}, \Psi_t\left(R_t = R^h, C_t = C^l, \Psi_{t-1} = 1\right)\right) \middle| N_t = N^h \right]. \end{aligned}$$

Since J^{PR} is upward sloping in reputation Ψ_t, an equivalent condition is

$$\Psi_t\left(R_t = R^h, C_t = C^h, \Psi_{t-1} = 0\right) \geq \Psi_t\left(R_t = R^h, C_t = C^l, \Psi_{t-1} = 1\right).$$

When $\lambda = 1$, the right-hand side is zero and the left-hand side is positive, so the inequality strictly holds. By continuity of these functions, for λ close enough to one the inequality will continue to hold.

C.9. Proof of Lemma 3

The necessary condition (4.16) can be rewritten

$$\frac{(1-q)\,p\lambda}{(1-q)\,p\lambda+q\,[p\lambda+(1-p)\,(1-\lambda)]}\geq\frac{qp\,(1-\lambda)}{qp\,(1-\lambda)+(1-q)\,[p\,(1-\lambda)+(1-p)\,\lambda]}.$$

Some algebra leads to the quadratic equation

$$\lambda^2\,(1-2p)\,(1-2q)+\lambda\left(p-2pq+2q^2-2pq^2\right)+(p-1)\,q^2\geq 0.$$

The positive root of this quadratic inequality defines $\lambda_{\min}\,(p,q)$.

C.10. Proof of Theorem

To prove the theorem, we use the three lemmas detailed in Section 4.4, as well as three additional lemmas that we now provide.

First denote the lower bound on the set of recurrent belief states:

$$\Psi^{\inf}\equiv\liminf_{t\to\infty}\Psi_t^h.$$

We show:

Lemma 4. *The value of $\Psi^{\inf}$ is determined as the fixed point to the equation*

$$\Psi_t=\Psi\left(R_t=R^h,C_t=C^l,\Psi_{t-1}\right), \tag{C.9}$$

where the analytical formula for the belief updating function $\Psi\,(.)$ is provided in the proof of Proposition 5 by equation $(C.2)$. Solving this fixed point, $\Psi^{\inf}$ is given by the negative root of the quadratic equation

$$(2q-1)\lambda\,(p-1)\,\Psi_t^2+[2p\,(1-\lambda)\,(1-q)+(1-p)\,q\lambda]\,\Psi-p\,(1-\lambda)\,(1-q)=0 \tag{C.10}$$

with coefficients in p, q, and λ.

Proof. For any Ψ_{t-1}, inspection of $(C.2)$ and $(C.3)$ shows that the minimum value of $\Psi\,(R_t,C_t,\Psi_{t-1})$ occurs when $R_t=R^h$ and $C_t=C^l$. Thus, the infimum of the set $\{\Psi:\Psi_t=\Psi$ infinitely often$\}$ is obtained as the limit of the recursion $(C.9)$. We solve the fixed point of this equation by substituting the belief updating formula $(C.2)$ on the right hand side. Some algebra gives $(C.10)$ ■

By the defition of $\Psi^{\inf}$, belief states $\Psi<\Psi^{\inf}$ are transient. One can quickly verify that if beliefs Ψ_0 are initially in the transient region, the progression to the recurrent set is monotonic: i.e., $\mathbb{P}\left(\Psi_{t+1}>\Psi_t\middle|\,\Psi_t<\Psi^{\inf}\right)=1$. Further, $\mathbb{P}\left(\Psi_{t+s}\leq\Psi^{\inf}\middle|\,\Psi_t>\Psi^{\inf}\right)=0$

for all $s > 0$, which ensures that once beliefs enter the recurrent region, they remain in it. We then note:

Lemma 5. *When $\lambda = 1$, then $\Psi^{\inf} = 0$. When $\lambda < 1$, then $\Psi^{\inf} > 0$.*

Proof. These results are obtained immediately from the solution to $(C.10)$ ■

One can also verify that $\Psi^{\inf}$ increases in λ and decreases in q by checking the derivative of the solution to $(C.10)$.

We further define the minimal recurrent wage under R^h declarations

$$W_h^{\inf} \equiv W^{PR}\left(R^h, \Psi^{\inf}\right) = \alpha^* \left(S^h - S^l\right) \frac{p}{p + (1-p)\left[q + (1-2q)\Psi^{\inf}\right]}.$$

Note that when $\lambda = 1$ then $W_h^{\inf} = W_h^{\min}$, otherwise $W_h^{\inf} > W_h^{\min}$. Also let

$$W^{\max} \equiv \max_{0 \le \Psi_{t-1} \le 1} \left[W^{PR}\left(R^h, \Psi_{t-1}\right)\right] = \alpha^* \left(S^h - S^l\right) \frac{p}{p + (1-p)(1-q)}.$$

The final preliminary result necessary to demonstrate the existence of equilibrium is to show how the reputation-building motive varies with the subjective discount rate β.

Lemma 6. *For fixed parameters (Θ, λ) and utility function u, and all states (N_t, S_t, Ψ_{t-1}), the reputation benefit Δ_J^{PR} increases in β, and*

$$\lim_{\beta \to 1} \Delta_J^{PR}(N_t, S_t, \Psi_{t-1}) < \infty.$$

Proof. Following equations (4.7) and (4.10), we obtain

$$\Delta_J^{PR}(N_t, S_t, \Psi_{t-1}) = \sum_{k=1}^{\infty} \beta^k \left[\bar{u}_{t+k}^l - \bar{u}_{t+k}^h\right] \tag{C.11}$$

where for $i \in \{l, h\}$,

$$\bar{u}_{t+k}^i \equiv \mathbb{E}\left[u\left(N_{t+k} + W^{PR}\left(R^{PR}(N_{t+k}, S_{t+k}), \Psi_{t+k-1}\right)\right) \middle| R_t = R^i, N_t, S_t, \Psi_{t-1}\right].$$

The difference $\bar{u}_{t+k}^l - \bar{u}_{t+k}^h$ is positive for all k, which can be confirmed by noting that a low declaration in period t will result in $\Psi_t = 1$, while a high declaration ensures $\Psi_t < 1$. Hence Δ_J^{PR} increases monotonically in β.

We now confirm that the limit as $\beta \to 1$ is finite. First define

$$\tau_l \equiv \min\left\{\tau > t \middle| N_\tau = N^h, S_\tau = S^l\right\},$$

which is the first time subsequent to t that the state $\left(N^h, S^l\right)$ arrives. Intuitively, declaring $R_t = R^l$ gives a reputation benefit with a stochastic life that is finite with probability one. For τ satisfying $t < \tau < \tau_l$, the declaration $R_t = R^l$ results in strictly higher reputation Ψ_τ than if the report $R_t = R^h$ had been made. For $\tau \geq \tau_l$ the report given at time t is irrelevant, because once R^l is declared again at time τ_l, reputation becomes one independent of prior reputation. Hence, we can rewrite $(C.11)$ as

$$\Delta_J^{PR}\left(N_t, S_t, \Psi_{t-1}\right) = \sum_{k=1}^{\infty} \beta^k \mathbb{P}\left(t+k < \tau_l\right) \left[\bar{u}_{t+k}^{l*} - \bar{u}_{t+k}^{h*}\right]$$

where for $i \in \{l, h\}$,

$$\bar{u}_{t+k}^{i*} \equiv \mathbb{E}\left[u\left(N_{t+k} + W^{PR}\left(R^{PR}\left(N_{t+k}, S_{t+k}\right), \Psi_{t+k-1}\right)\right) \middle| R_t = R^i, N_t, S_t, \Psi_{t-1}, t+k < \tau_l\right].$$

Observe that $\bar{u}_{t+k}^{l*} - \bar{u}_{t+k}^{h*}$ is bounded above by a constant that we denote $\Delta_{\bar{u}}^{\max}$. Further, $\mathbb{P}\left(t+k < \tau_l\right) \leq \varphi^k$, where $0 < \varphi \equiv 1 - (1-q)(1-p) < 1$. Hence,

$$\Delta_J^{PR}\left(N_t, S_t, \Psi_{t-1}\right) \leq \sum_{k=1}^{\infty} \beta^k \varphi^k \Delta_{\bar{u}}^{\max}.$$

When $\beta = 1$, the right-hand side remains finite because of the geometric decline in φ^k. By the dominated convergence theorem, Δ_J^{PR} must converge as well, and the result is established.■

The previous lemma may seem somewhat surprising since the intertemporal utility J^{PR} itself diverges to ∞ as $\beta \to 1$. The quantity Δ_J^{PR} is however the present value of the differential future utility benefit between declaring R^l and R^h. Intuitively, the reputation benefit from declaring R^l may randomly terminate in each future period if the state $\left(N^h, S^l\right)$ occurs again, and this probability acts as an additional discount factor (whose upper bound is given by φ^k in the proof), which keeps Δ_J^{PR} finite even as $\beta \to 1$.

Using Lemmas 1-6, we now complete the proof of existence of equilibria by constructing utility functions that satisfy the PR equilibrium conditions as required by the Theorem. The utility functions $u(c)$ can be constructed as follows:

i) Choose the function $u(c)$ to be sufficiently flat for $c \geq N^h + W_h^{\min}$. Denote the average slope $s_{hi} \equiv \left[u\left(N^h + W^{\max}\right) - u\left(N^h + W_h^{\min}\right)\right] / \left(W_h^{\min} - W^{\max}\right)$ and choose this close to zero.

ii) Choose the utility difference over the interval N^h to $N^h + W_h^{\min}$ to be strictly positive. Let $s_{mid} \equiv \left[u\left(N^h + W_h^{\min}\right) - u\left(N^h\right)\right] / W_h^{\min} > 0$. If $s_{hi} = 0$, then

$\Delta_u^{PR}\left(N^h, 0\right) = \Delta_u^{PR}\left(N^h, 1\right) = s_{mid} W_h^{\min}$, and the conditions $C1'$ and $C2'$ (alternatively, equation (C.6)) reduce to

$$\Delta_J^{PR}\left(N^h, S^h, 0\right) \leq s_{mid} W_h^{\min} \leq \Delta_J^{PR}\left(N^h, S^l, 1\right). \tag{C.12}$$

Lemma 2 establishes that for λ close to one the relation between the endpoints is satisfied.

iii) The average slope over the interval $N^l + W_h^{\inf}$ to N^h must be sufficiently large relative to s_{mid}. Specifically, let

$$s_{lo} \equiv \left[u\left(N^h\right) - u\left(N^l + W_h^{\inf}\right)\right] / \left[N^h - \left(N^l + W_h^{\inf}\right)\right].$$

Condition $C\Theta$ guarantees that for λ close to one $N^h - \left(N^l + W_h^{\inf}\right) > 0$, and so we can freely increase the average slope s_{lo} without affecting s_{mid} or s_{hi}. Then, holding constant the upper portion of the utility function ($c \geq N^h$) and the parameters Θ and λ, choose the lower slope s_{lo} sufficiently large to increase the reputation motive and satisfy

$$s_{mid} W_h^{\min} < \lim_{\beta \to 1} \Delta_J^{PR}\left(N^h, S^l, 1\right). \tag{C.13}$$

If (C.13) holds, it becomes immediately clear that (C.12) will be satisfied for a range of $\beta \in (0, 1)$.

As an aside, we note that the slope of u over the entire range $N^l + W_h^{\inf}$ to N^h is not critical, and increasing marginal utility in the neighborhood of $N^l + W_h^{\inf}$ is sufficient sufficient to satisfy (C.13). Specifically, for any constant δ satisfying $0 < \delta \leq N^h - \left(N^l + W_h^{\inf}\right)$, we can hold constant the function u above $N^l + W_h^{\inf} + \delta$, and increase the slope over the range $\left[N^l + W_h^{\inf}, N^l + W_h^{\inf} + \delta\right]$ until (C.13) holds.

iv) Choose the average slope over the interval N^l to $N^l + W_h^{\min}$ to be sufficiently large relative to s_{lo}. Let $s_0 \equiv \left[u\left(N^l + W_h^{\min}\right) - u\left(N^l\right)\right] / W_h^{\min}$. The value s_0 can be chosen arbitrarily large, ensuring that $\Delta_u^{PR}\left(N^l, 0\right)$ is sufficiently large that $C3'$ holds, without affecting any of the quantities in $C1'$ or $C2'$.

Using this construction, it is clear that we can construct an arbitrarily large family of functions that satisfy the equilibrium conditions for λ close to one.

As a commentary outside the proof, we note that for a *fixed* utility function $u(c)$, the equilibrium conditions may be satisfied for a range of λ strictly less than one, while no equilibrium exists for λ near one. (Cases like this can be observed in Panels A, C, and E of Figure 4). When this occurs it is because condition $C2'$ is violated for λ near one, and even for β approaching one the reputation benefit is not strong enough relative to current income concerns. Lower values of λ make the value of reputation larger since it takes longer to learn about the manager's true type, and hence it is possible that for fixed utility function $u(c)$ equilibria can exist for λ strictly less than one while no equilibria exist near one. In such cases, equilibria can be found for λ near one by altering the utility function to make it more concave (as in part iii of the construction above), which also strengthens the reputation motive.

C.11. Proof of Proposition 7

Under FR, the manager's wage must be flat $W_t = W_0$ to efficiently allocate risk. Thus, when the manager declares $R_t = R^i$, $i \in \{h, l\}$, investors plan for aggregate consumption $\widehat{\gamma}_t = C^i - W_0$. Expected recontracting costs are therefore $\mathbb{E}_{FR}(\xi_t | \Phi_{t-1}) = (1-\lambda)\xi\Delta_C$ where $\Delta_C \equiv C^h - C^l$.

Under PO, the manager's report is irrelevant and investors will always plan for the consumption level that has the highest unconditional probability. Given that $\mathbb{P}(C^h) = 1 - \lambda - p + 2\lambda p$ and $\mathbb{P}(C^l) = \lambda + p - 2\lambda p$, expected recontracting costs $\mathbb{E}_{PO}(\xi_t | \Phi_{t-1})$ are equal to the lower of $(1 - \lambda - p + 2\lambda p)\xi\Delta_C$ and $(\lambda + p - 2\lambda p)\xi\Delta_C$. Either value is strictly higher than under FR.

Under PR, investors always plan for consumption $\widehat{\gamma}_t = C^l - W(R^l)$ when $R_t = R^l$ is declared. If when $R_t = R^h$, investors also plan for consumption $\widehat{\gamma}_t = C^h - W(R^h)$, then $\mathbb{E}(\Pi_t^h | \Phi_{t-1}) = 1 - \mathbb{E}(\Pi_t^l | \Phi_{t-1}) = \Psi_{t-1}^h q + \Psi_{t-1}^l (1-q)$ is the probability that the manager will have high non-wage income at time t given what investors know at the end of period $t-1$. This leads to the conclusion that

$$\mathbb{E}_{PR}(\xi_t | \Phi_{t-1}) = \left[\mathbb{E}\left(\Pi_t^h | \Phi_{t-1}\right)(1-\lambda) + \mathbb{E}\left(\Pi_t^l | \Phi_{t-1}\right)(p + \lambda - 2\lambda p)\right]\xi\Delta_C.$$

Alternatively, if when $R_t = R^h$ investors plan for consumption $\widehat{\gamma}_t = C^l - W(R^h)$, then expected recontracting costs would be equal to $(1 - \lambda - p + 2\lambda p)\xi\Delta_C$. Investors thus choose the lower of these two values under PR. We conclude that $\mathbb{E}_{FR}(\xi_t | \Phi_{t-1}) \leq \mathbb{E}_{PR}(\xi_t | \Phi_{t-1}) \leq \mathbb{E}_{PO}(\xi_t | \Phi_{t-1})$.

C.12. Proof of Proposition 8

For any pooling equilibrium with wage sensitivity α^* and wage $W_0 = \alpha^* \mathbb{E}(C_t - S^l)$, consider the separating equilibrium with the identical flat wage. The manager is paid the same, but expected recontracting costs are lower under FR.

D. Equilibrium Conditional on Initial Beliefs

In section 4 we provide conditions under which the PR equilibrium exists for any choice of initial beliefs Ψ_0. Given a specific choice of Ψ_0, the incentive compatibility conditions can potentially be weakened, as we now discuss.

This argument begins with Lemma 5, which show that if $\lambda < 1$, then $\Psi^{\inf} > 0$. (See Proof of Theorem, Appendix C.) The discussion of Lemma 4 also shows that if initial beliefs Ψ_0 are less than $\Psi^{\inf}$, then beliefs will progress monotonically upward until they are in the recurrent region where $\Psi > \Psi^{\inf}$. Hence, the requirement of Bayesian updating in Markov perfect Bayesian equilibrium ensures that the minimum belief level achieved on any path is $\Psi^{\min} \equiv \min\left(\Psi_0, \Psi^{\inf}\right)$. For a given value of Ψ_0, the incentive compatibility conditions given in Proposition 6 can therefore be weakened as follows:

$$\begin{aligned} C1': \quad & \Delta_u^{PR}\left(N^h, \Psi^{\min}\right) \geq \Delta_J^{PR}\left(N^h, S^h, \Psi^{\min}\right) \\ C3': \quad & \Delta_u^{PR}\left(N^l, \Psi^{\min}\right) \geq \Delta_J^{PR}\left(N^l, S^l, \Psi^{\min}\right). \end{aligned}$$

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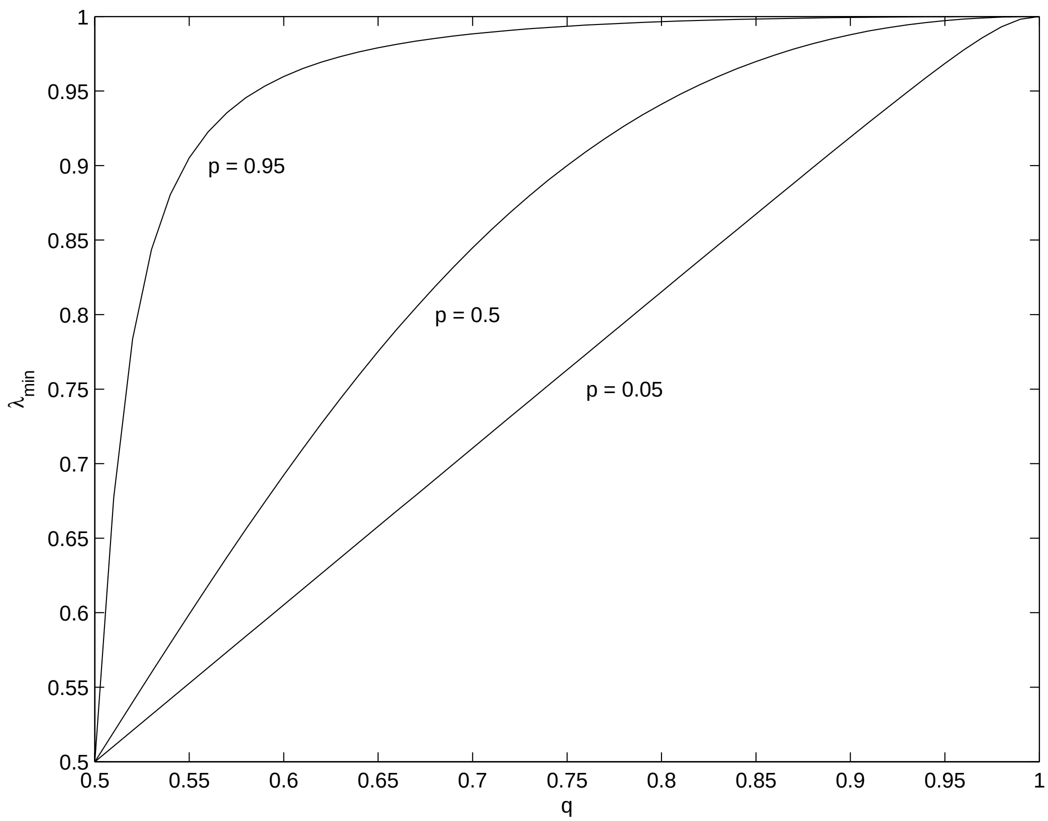

Figure 1: Minimum Information Precision in the PR Equilibrium. This figure shows the minimum values of the information precision parameter λ such that the PR equilibrium may hold. For $p \in \{0.05, 0.5, 0.95\}$, the three plots show how λ_{min} varies with q. The minimum value decreases in p and increases in q, consistent with Lemma 3.

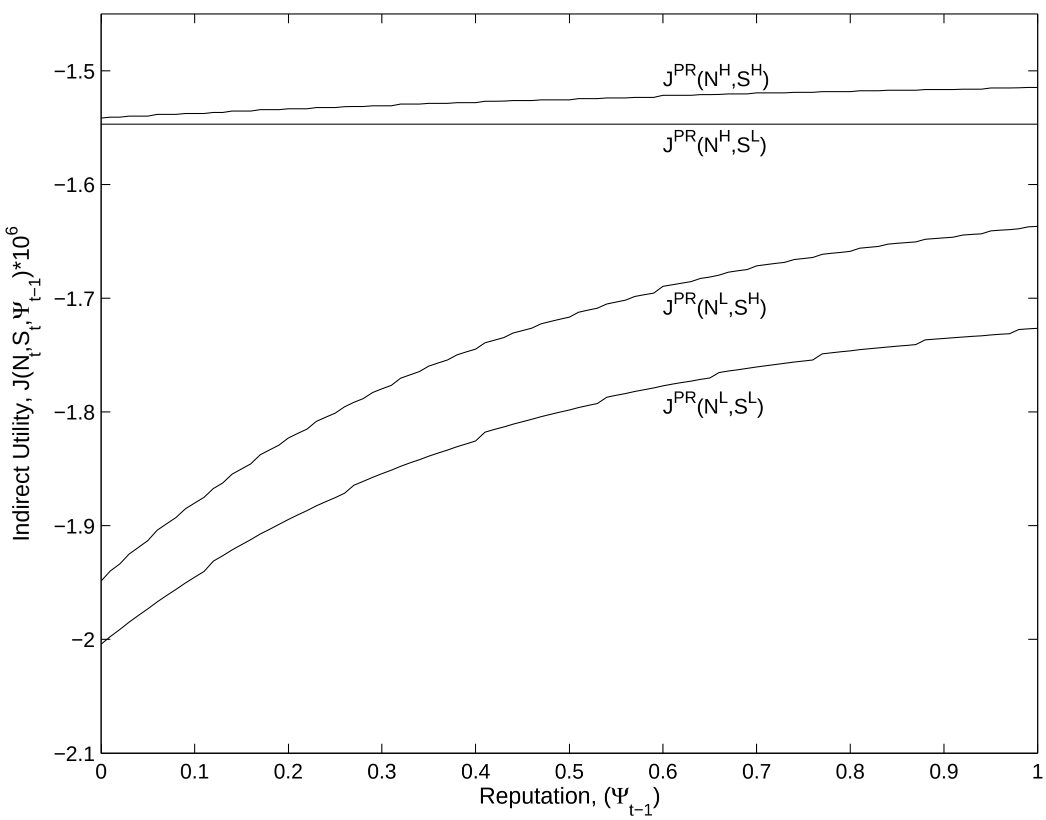

Figure 2: The Derived Utility of the Partial Revelation (PR) Policy. This figure shows the manager's derived utility under the PR strategy. The figure uses the base parameters $p = 0.5$, $\lambda = 0.9$, $\alpha* = 0.037$, $\gamma = 10$, $\beta = 0.95$, $q = 0.75$, $N^l = 0.01$, $N^h = 5$, $C^l = 0$, and $C^h = 250$.

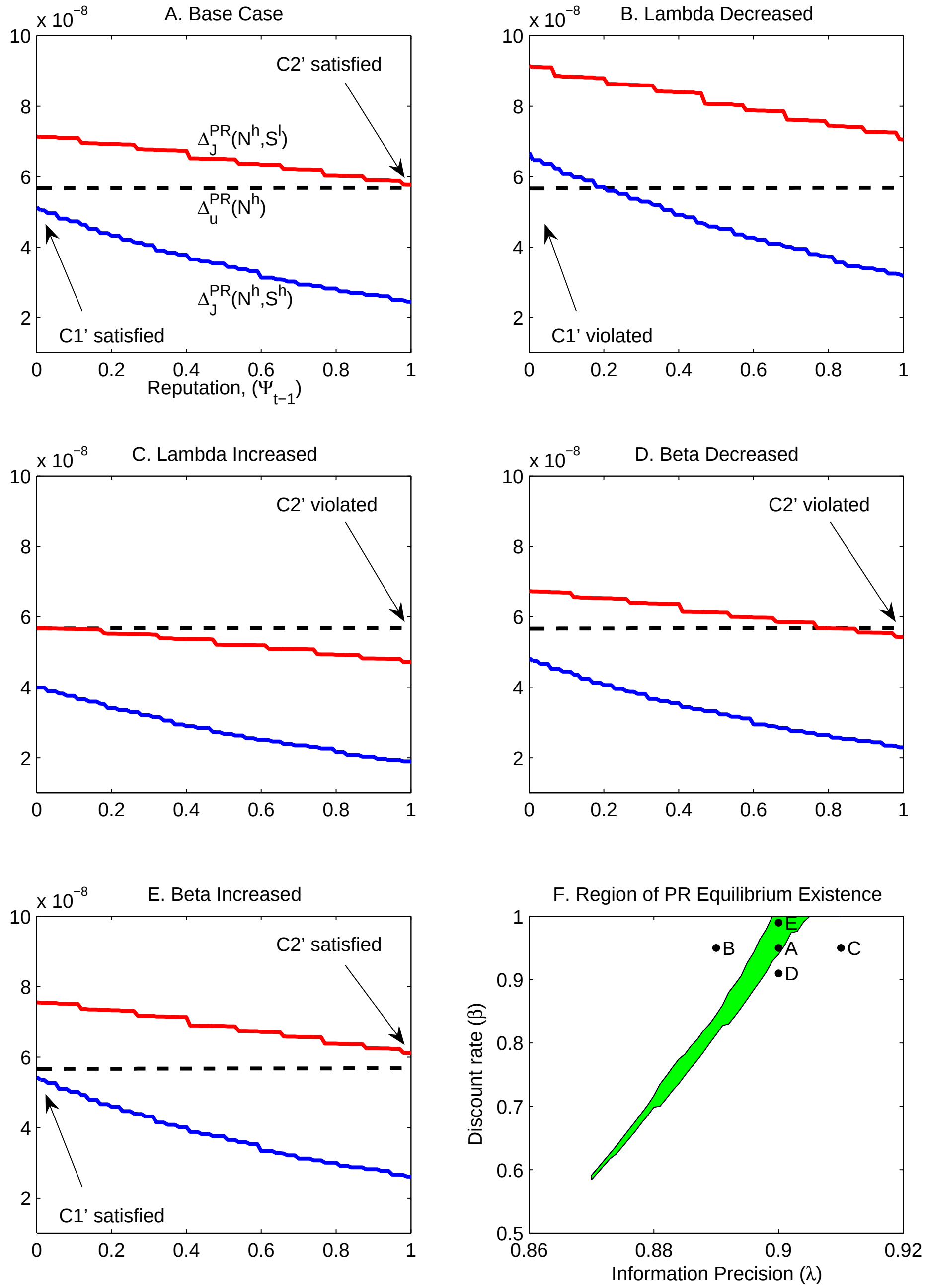

Figure 3: Incentive Compatibility in the PR Equilibrium. Panels A-E show the functions $\Delta_J^{PR}(N^h, S^j)$ for $j \in \{h, l\}$ and $\Delta_u^{PR}(N^h)$. Panel A uses the base parameters described in the text and notes to Figure 2. Panel B decreases information precision to $\lambda = 0.89$ and Panel C increases it to $\lambda = 0.91$. Panels D and E change the subjective discount rate to $\beta = 0.91$ and $\beta = 0.99$ respectively. Panel F shows the full range of existence of the PR equilibrium in (λ, β) space. The letters in Panel F indicate the position of the parameters in Panels A-E.

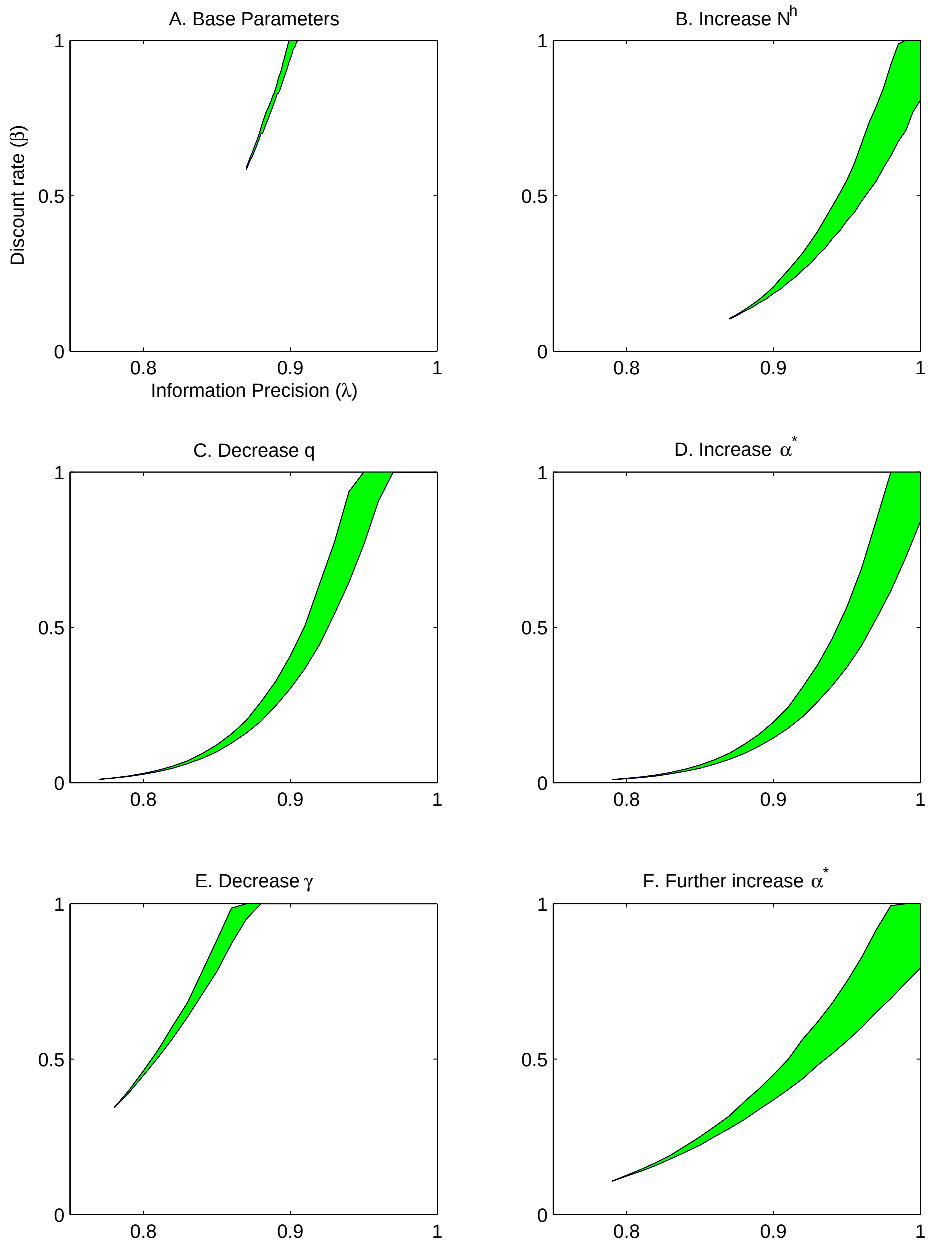

Figure 4: Regions of Existence of the PR Equilibrium. This figure shows the regions of the (λ, β) space where the PR equilibrium holds for different values of the other parameters. Panel A uses the base parameters detailed in the text and notes to Figure 2. Each consecutive panel recursively holds constant all parameters from the previous panel except one, which is modified as now explained. Panel B increases the high level of non-wage income to $N^h = 6.2$. Panel C decreases the persistence of non-wage income to $q = 0.6$. Panel D decreases pay sensitivity to $\alpha^* = 0.034$. Panel E decreases risk aversion to $\gamma = 5$, and Panel F further decreases pay sensitivity to $\alpha^* = 0.025$.

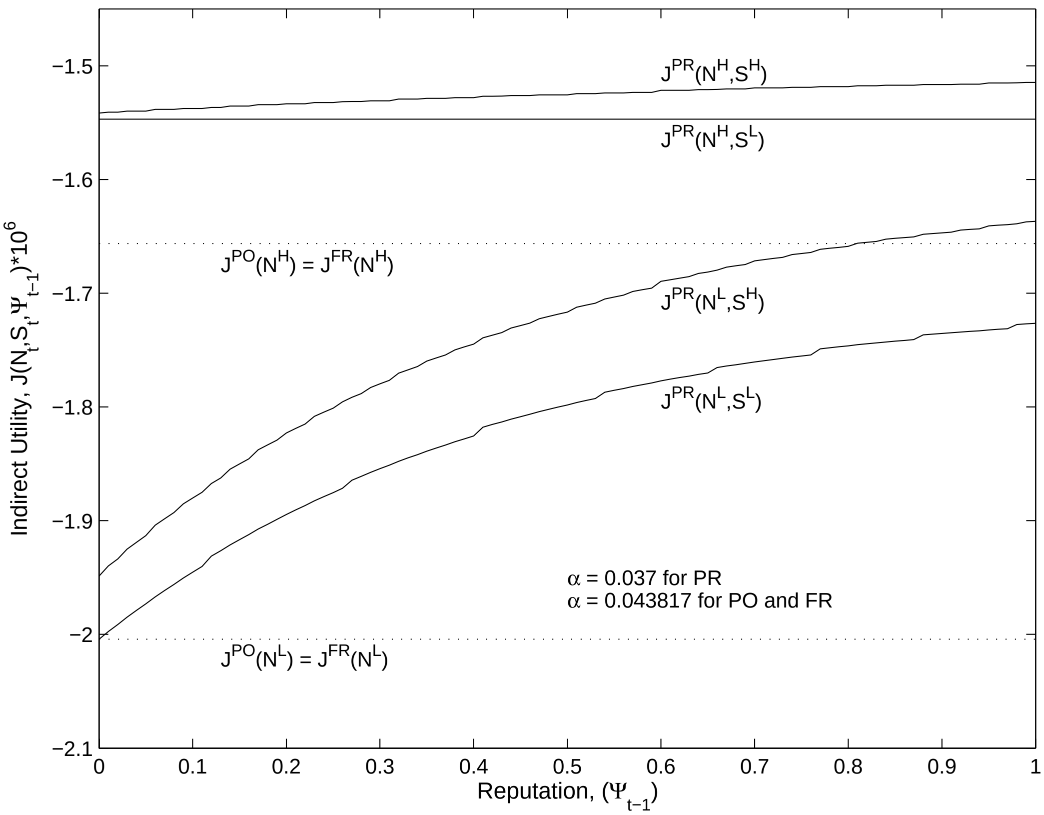

Figure 5: Indirect Utilities under Partial Revelation (PR), Full Revelation (FR) and Pooling (PO) Equilibria. This figure shows the manager's indirect utility under PR (solid lines) and FR and PO equilibria (dotted lines). The paramaters used for the PR equilibrium are the base parameters used in Figure 2, where $\alpha_{PR} = 0.037$. To determine $\alpha_{PO} = \alpha_{FR} = 0.043817$, we increase wage sensitivity until a state exists where the manager is indifferent between PR and the FR and PO equilibria. Even though the manager's average wage is higher in the pooling and full revelation equilibria, he is at least as well off in every state under PR, due to the self-insurance effect of manipulating wages.

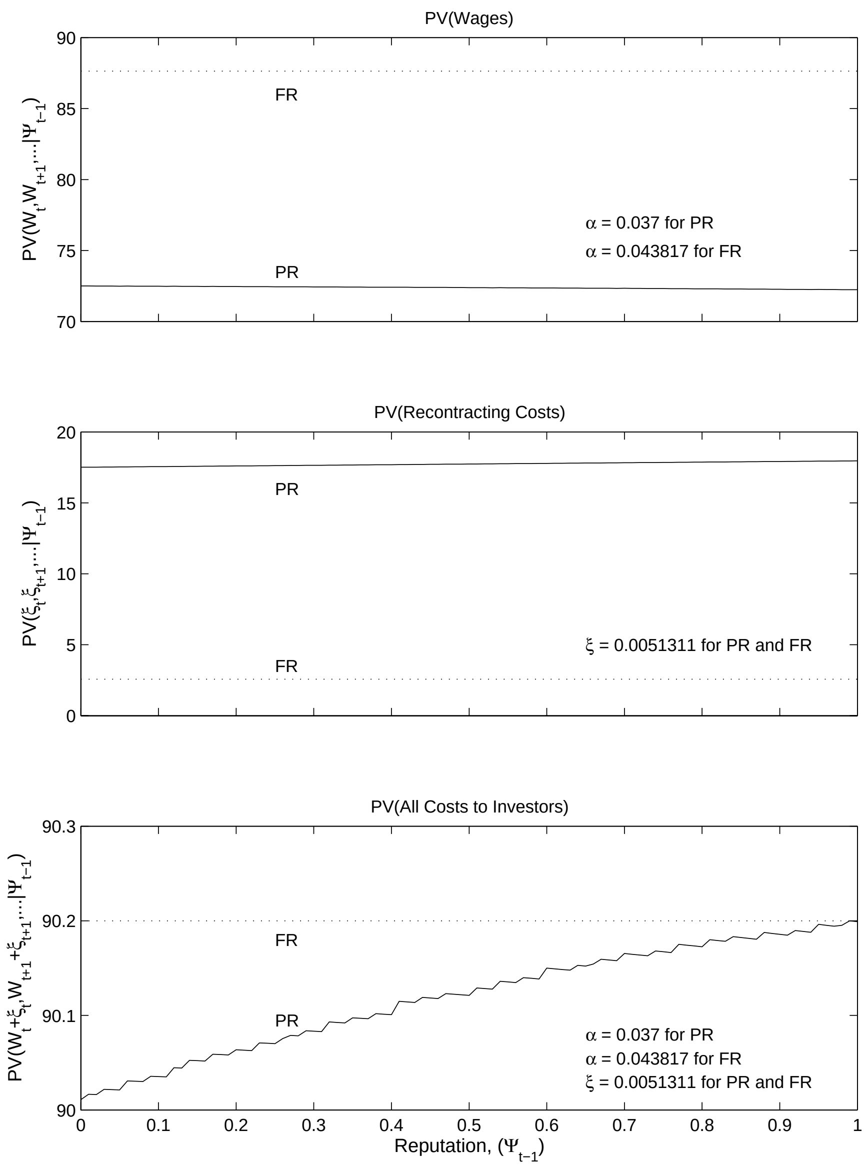

Figure 6: Costs to Investors under Partial Revelation (PR) and Full Revelation (FR) Equilibria. This figure shows the present value of costs to investors under PR (solid lines) and FR (dotted lines). The first panel shows the wage component, the second the recontracting costs, and the third panel gives the combined effects. The parameter $\alpha_{PR} = 0.037$ is from the base parameter specification, and $\alpha_{FR} = 0.0438$ was derived in Figure 1 to give the corresponding maximum FR wage sensitivity in which the manager remains weakly better off in every state of the world under PR. We then examine the welfare of investors. Specifically, we derive the maximum recontracting cost ξ for which investors are better off in every state of the world under PR. PR interim Pareto dominates FR whenever aggregate recontracting costs are below approximately one half percent of firm value.