

# Leaders, Followers, and Risk Dynamics in Industry Equilibrium\*

Murray Carlson, Engelbert J. Dockner, Adlai Fisher,  
and Ron Giammarino

April 28, 2009

## Abstract

We study own and rival risk in a dynamic duopoly with a homogeneous output good, stochastic industry demand, real options to expand or contract capacity, and potentially different adjustment costs across firms. In general, a competitor's options to adjust capacity reduce own-firm risk through a simple hedging channel. When a rival possesses a growth option, an increase in industry demand directly enhances current profits and value, but also increases the prospect of value-reducing competitor expansion. As the rival moves closer to its expansion or contraction boundaries, such hedging effects become more important and generally differ from the own-firm effects of real options. As a consequence, when a leader and a follower emerge in equilibrium, risk dynamics depart substantially from a simultaneous move benchmark. In leader-follower equilibria own-firm and competitor required returns

---

\*Carlson, Fisher, and Giammarino: Sauder School of Business, University of British Columbia. Dockner: Vienna University of Economics and Business and Vienna Graduate School of Finance. We received helpful comments from seminar participants at the Copenhagen School of Business, Duke University, HEC Lausanne, Mannheim University, Queensland University of Technology, Texas A&M, Washington University, the University of Amsterdam, the University of Bern, the University of Calgary, the University of Graz, the University of New South Wales, the University of Texas at Dallas, the 2006 Meetings of the German Finance Association, the 2006 UBC PH&N Summer Conference, the 2007 Meetings of the American Finance Association, the 2007 Meetings of the Northern Finance Association, the 2008 Corporate Finance Conference at Wilfrid Laurier University, the 2008 European Winter Finance Summit, and the 2008 North American Summer Meeting of the Econometric Society. The financial support of the Social Sciences and Humanities Research Council of Canada is gratefully acknowledged.

tend to move together through contractions and oppositely during expansions. Thus, the common practice of using industry peer betas to proxy for own-firm risk should work well in certain environments, but not in others, providing testable new empirical predictions.

# Leaders, Followers, and Risk Dynamics in Industry Equilibrium

## Abstract

We study own and rival risk in a dynamic duopoly with a homogeneous output good, stochastic industry demand, real options to expand or contract capacity, and potentially different adjustment costs across firms. In general, a competitor's options to adjust capacity reduce own-firm risk through a simple hedging channel. When a rival possesses a growth option, an increase in industry demand directly enhances current profits and value, but also increases the prospect of value-reducing competitor expansion. As the rival moves closer to its expansion or contraction boundaries, such hedging effects become more important and generally differ from the own-firm effects of real options. As a consequence, when a leader and a follower emerge in equilibrium, risk dynamics depart substantially from a simultaneous move benchmark. In leader-follower equilibria own-firm and competitor required returns tend to move together through contractions and oppositely during expansions. Thus, the common practice of using industry peer betas to proxy for own-firm risk should work well in certain environments, but not in others, providing testable new empirical predictions.

*JEL Classification:* C23, C35.

*Keywords:* Growth options and industry risk, asset pricing and investment decisions, risk dynamics in oligopolistic industries

# 1 Introduction

A corporation's opportunities to expand, contract, or otherwise alter production can impact its risk and return dynamics, as observed by Berk, Green, and Naik (1999), Gomes, Kogan, and Zhang (2003), and subsequent authors.<sup>1</sup> In an industry setting, a firm's decisions may additionally affect the required returns of product market rivals, and vice versa. Identifying the distinct impacts of own and rival real options on firm risk can be useful to both finance research and practice. For example, financial analysts often estimate the required return of a product or corporation using not only the historical risk of the firm, but also its industry rivals.<sup>2</sup> Theory may therefore help to shed light on practice by explaining how competitor real options impact own-firm risk as industry conditions change, and conversely how own-firm real options affect the required returns of rivals.

In this paper, we study own and rival risk in a dynamic duopoly with a homogeneous output good, real options to expand or contract capacity as industry demand changes, and potentially different adjustment costs across firms. For some parameter values the unique equilibrium requires firms to exercise their options simultaneously, implying that the expected returns of the two firms move together as in the symmetric oligopoly studied by

---

<sup>1</sup>See, for example, Aguerrevere (2009), Carlson, Fisher, and Giammarino (2004, 2006), Cooper (2006), Hackbarth and Morellec (2008), Johnson (2002), Kogan (2004), Sagi and Seasholes (2007), and Zhang (2005). This literature builds on the seminal work of Brennan and Schwartz (1985) and McDonald and Siegel (1985, 1986) and is discussed in more detail below.

<sup>2</sup>The widely used Ibbotson *Beta Book* provides estimates of beta based on a peer group that depends on industry classification, and the use of industry competitors to proxy for own-firm risk is discussed in finance textbooks such as Brealey and Myers (2001), and Ross, Westerfield, and Jaffe (1996).

Aguerrevere (2009). By contrast, even arbitrarily small differences in adjustment costs can imply non-simultaneous exercise in which one firm acts as a leader and the second as a follower. In such cases, the risk of a firm and its rival may alternately move together or apart over time, depending on industry conditions and the corresponding changing importance of own and rival growth and contraction options. Because of these dynamics, in a leader-follower equilibrium the joint evolution of required returns departs substantially from simultaneous move outcomes.

For both expansions and contractions, we show that rival real options reduce own-firm risk through a simple hedging channel. For example, when a competitor possesses a growth option any good news about the product market is partially offset by the closer proximity of rival expansion. Conversely, bad news about industry demand is counterbalanced by a decline in the threat of competitor capacity additions. Hence, all else equal rival growth options reduce own firm risk. Similarly, when the rival possesses a contraction option, industry demand shocks are partially offset by opposite movements in the likelihood of near-term rival asset sales, again reducing own firm risk. The magnitudes of the hedging effects created by rival real options change over time with industry conditions and the distance to the competitor's option exercise boundaries.

To develop intuition in the simplest case possible, we first consider an industry where one firm is a "strategic dummy" with permanently fixed output, while the second firm has a single option to irreversibly expand or contract its quantity supplied. At each instant, prices are determined by aggregate industry output and both firms receive a flow of profits. The

firm possessing an option to adjust capacity has upper and lower bounds for expansion and contraction, and risk dynamics similar to those shown in prior literature focusing on monopolist exercise. Although the strategic dummy has no real options of its own, its valuation equations and risk nonetheless reflect the dynamic output policies of its rival. In particular, the risk of the strategic dummy decreases as its rival moves towards either its expansion or contraction boundary, and immediately jumps up to a constant when the rival exercises its option.<sup>3</sup>

In the more general case where both firms may expand or contract, the own-firm and rival valuation equations and betas can possess up to four real options components. Whether on an expansion or a contraction path, the leader's real option tends to be more important for both own and rival valuation equations and risk, because the follower option is further out-of-the money. On an expansion path, the dynamics of leader and follower risk follow a distinctive pattern. As the leader moves closer to exercise, her own risk increases due to growth option leverage, while the follower risk decreases due to the rival hedging effect. Immediately at the instant the leader exercises her growth option, the risks of the two firms jump oppositely by sufficient magnitudes such that the follower risk exceeds leader risk. Thus, on a path of increasing industry demand own- and rival-firm risk tend to move in opposite directions. By contrast, the own-firm and rival-firm effects of contraction options have the same sign, and in an environment of decreasing industry demand the leader and follower risks tend to move together.

---

<sup>3</sup>The discontinuity in risk at the instant of competitor option exercise reflects the generic lack of smooth pasting when other players take discrete actions in a continuous-time game.

These theoretical results suggest that the commonly recommended practice of using competitor or industry betas to proxy for own-firm risk should work well in certain environments, but not in others, providing testable new empirical predictions.

Our paper builds on several areas of the literature. Berk, Green, and Naik (1999) and Gomes, Kogan, and Zhang (2003) model real options by their cash flows and discount rates, abstracting from explicit consideration of the product market. Subsequent literature considers risk dynamics due to real options in a variety of homogeneous goods market structures. Carlson, Fisher, and Giammarino (2004, 2006) and Cooper (2006) study cross-sections of monopolists with varying options to expand, contract, enter, or exit. Kogan (2004) analyzes risk and return in a perfectly competitive industry with symmetric firms and investment irreversibility, while Zhang (2005) allows cross-sectional firm differences within a perfectly competitive industry. Aguerrevere (2009) builds on Aguerrevere (2003) and Grenadier (2002) to analyze risk and return in a symmetric simultaneous-move oligopoly.<sup>4</sup>

Our work differs from the prior finance literature by permitting asymmetric adjustment costs in an oligopoly setting, which leads to leader-follower

---

<sup>4</sup>Other related work includes Novy-Marx (2008), who considers a simultaneous-move equilibrium in an oligopoly related to the Grenadier setting but where firms have cost differences; Hackbarth and Morellec (2008) who study risk dynamics in a merger setting; Carlson, Fisher, and Giammarino (2008) and Kuehn (2008), who discuss the impact of investment commitment on risk; and Pastor and Veronesi (2008) who discuss the impact of technological innovation on asset price dynamics. In a recent study, Back and Paulson (2009) discuss subgame perfection in continuous-time games where firms possess singular controls such as trigger decisions. They emphasize settings related to the Grenadier model where firms have unlimited expansion opportunities and in a closed-loop equilibrium all real option values are competed away. All of the equilibria we consider satisfy subgame perfection and hence form closed-loop equilibria, but option values remain positive because expansion opportunities are finite.

risk dynamics in some cases, and simultaneous exercise in others. Under a leader-follower equilibrium characterized by sequential exercise, both own firm and rival options are important to valuations and have different implications for risk. Further, the relative importance of own- and rival- options shifts through time as industry conditions change, highlighting a new channel through which firms within the same industry may differ in their risk exposures.

Prior research in the real options literature analyzes equilibrium exercise of expansion or contraction opportunities in a duopoly setting, but does not investigate risk dynamics. Examples include Smets (1991), Dixit and Pindyck (1994), Huisman and Kort (1999), Boyer, Lasserre, Mariotti, and Moreaux (2001), and Murto (2004). In general, simultaneous exercise of growth options may occur even when firms have asymmetric adjustment costs, provided assets in place exist (e.g., Pawlina and Kort, 2006). Our framework emphasizes the importance of the product market demand elasticity in determining the boundary between simultaneous-exercise equilibrium and leader-follower equilibria. For high demand elasticities, simultaneous exercise can be supported for a large range of asymmetries in adjustment costs. By contrast, when demand elasticities are low, even arbitrarily small adjustment cost asymmetries can lead to leader-follower exercise as the unique equilibrium outcome.<sup>5</sup> For contraction options, no simultaneous-

---

<sup>5</sup>For many combinations of low demand elasticities and large growth options, even perfectly symmetric firms cannot optimally exercise growth options simultaneously, and a randomly chosen leader arising from mixed strategies is the only possibility. Huisman and Kort (1999) and Boyer, Lasserre, Mariotti, and Moreaux (2001) discuss mixed strategies in expansion games, which requires an extension of the strategy space beyond simple state-dependent triggers following Fudenberg and Tirole (1985). See also Dixit and Pindyck (1994), who discuss a symmetric duopoly where firms have no assets in place, following

move equilibria exist.

In all leader-follower equilibria<sup>6</sup> for both expansions and contractions, the distance between leader and follower triggers remains bounded below even for arbitrarily small adjustment cost asymmetries, or in the case of identical firms when the leader is randomly chosen. Intuitively, the leader's action, whether expansion or contraction, strategically impacts the incentives of the follower to create a finite separation in their actions. Hence, non-simultaneous exercise can be an important feature of both expansions and contractions, even when firms are ex ante very similar or identical. The risk dynamics that we demonstrate for leader-follower equilibria therefore fill an important gap in the finance literature.

Section 2 describes the general model. In Section 3, we analyze the simplest case where one firm is a strategic dummy, and show the risk-reducing effects of rival growth options. Section 4 presents the leader-follower equilibrium where firms with asymmetric costs may expand or contract. Section 5 concludes.

---

Smets (1991).

<sup>6</sup>Our notion of a "leader-follower" equilibrium is synonymous with "non-simultaneous." Several types of leader-follower equilibria are distinguished below and have been discussed in prior literature. In a "non-preemptive" equilibrium, the leader and follower use the trigger strategies that would arise if the follower were prohibited from acting first and the role of "leader" determined prior to the start of the game. In a "pre-emptive" equilibrium, the threat of action by the follower causes the leader to act earlier than she would if the rules of the game prohibited the follower from acting first. In a mixed strategy equilibrium, which may occur if firms are symmetric, the leader is determined randomly.

## 2 The Asymmetric Duopoly Model

We present a model in which two strategically interacting firms compete in output levels in a homogeneous goods market, and have options to invest or disinvest in capacity.

### 2.1 Industry Demand, Production Technologies, and Capital Accumulation

Let  $Q_t^1$  and  $Q_t^2$  denote the output rates of firm one and firm two at instant  $t$ , and define the industry output rate  $Q_t = Q_t^1 + Q_t^2$ . The homogeneous good price is determined by the iso-elastic inverse demand curve

$$P_t = X_t Q_t^{\gamma-1}, \quad (1)$$

where  $0 < \gamma < 1$ , and  $X_t$  is an exogenous state variable that represents the level of industry-wide demand. The dynamics of  $X_t$  are specified by

$$dX_t = gX_t dt + \sigma X_t dW_t, \quad (2)$$

where  $dW_t$  is the increment of a Wiener process,  $g$  is the constant drift, and  $\sigma^2$  the constant variance.

Firm  $i$  produces output at time  $t$  using installed capital  $K_t^i$  where  $i \in \{1, 2\}$ . Any capital level  $K_t^i$  is associated with a maximum output level  $Q(K_t^i) \geq Q_t^i$ . For simplicity, capital levels take one of three discrete values:  $K_t^i \in \{\kappa_0, \kappa_1, \kappa_2\}$ , where  $\kappa_0 < \kappa_1 < \kappa_2$ , and for convenience we denote  $q_j \equiv$

$Q(\kappa_j)$  with  $q_0 < q_1 < q_2$ .<sup>7</sup> Costs of production for firm  $i$  at date  $t$  are given by the increasing function  $F_t^i = f(K_t^i)$ . This cost structure emphasizes operating leverage, since total expenditures depend only on the installed capital level  $K^i$ , as with maintenance costs or other overhead related to plant size. Given the three possible capital levels, there are also three possible levels of fixed operating costs:  $F_t^i \in \{f_0, f_1, f_2\}$ , where  $f_0 < f_1 < f_2$ .

To move from one capital state to another, the firm may incur costs or generate cash flows from buying or selling the productive asset, inclusive of any associated adjustment costs. To capture this idea in a general way, we specify for each firm a matrix of discrete transition costs:

$$\Lambda^i \equiv \begin{bmatrix} 0 & \lambda_{01}^i & \lambda_{02}^i \\ \lambda_{10}^i & 0 & \lambda_{12}^i \\ \lambda_{20}^i & \lambda_{21}^i & 0 \end{bmatrix}.$$

The instantaneously incurred lump-sum cost for firm  $i$  to move from capital level  $\kappa_m$  to  $\kappa_n$  is given by  $\lambda_{mn}^i$ . The only source of heterogeneity between firms in our model is that  $\Lambda^1$  and  $\Lambda^2$  need not be identical. We assume as an initial condition that at date zero, each firm is endowed with  $K_0^i = \kappa_1$  units of capital.

We finally define indicator variables  $D_t^{imn}$  that take the value one at the instant when firm  $i$  switches from capital level  $\kappa_m$  to  $\kappa_n$ , and zero elsewhere.

We denote by  $D_t^i$  the matrix of investment decisions  $D_t^{imn}$ .

---

<sup>7</sup>The assumption that the potential output levels  $q_j$  are the same for firms 1 and 2 is not essential, and is made here for notational convenience. The arguments in the Appendix are valid when the output levels  $q_j$  differ across firms  $i$ , hence permitting asymmetric revenue functions.

## 2.2 Output, Investment Strategies, and Equilibrium

The economy described above is a dynamic game between firms 1 and 2. At each instant, the managers of the two firms choose output rates  $Q_t^i$  and make investment decisions  $D_t^i$  knowing the complete history of the game denoted by  $\Phi_t = \left( [Q_s^1, Q_s^2, K_s^1, K_s^2]_{s < t}, [X_s]_{s \leq t} \right)$ , which is common to both managers.

We define the payoff to firm  $i$  as the present value of the expected discounted future cash flows. The cash flows at time  $t$  derive from revenues in excess of fixed costs  $\pi_t^i \equiv P_t Q_t^i - F_t^i$  and from lumpy investment costs related to the decision  $D_t^i$ . We assume the absence of agency conflicts, so that manager  $i$  maximizes the value function

$$V_t^i \equiv E_t \int_t^\infty e^{-r(s-t)} \frac{M_{t+s}}{M_t} [\pi_{t+s}^i ds + \mathbf{1}' (D_{t+s}^i * \Lambda^i) \mathbf{1}], \quad (3)$$

where  $\mathbf{1}' = [1, 1, 1]$ ,  $*$  represents element-by-element multiplication, and the pricing kernel  $M_t$  satisfies  $M_0 = 1$  and  $dM_t = \frac{\mu-r}{\sigma} M_t dW_t$ .

Given the Markov structure of this environment, it is natural to restrict attention to Markov strategies. Manager  $i$  can then take actions  $Q_t^i$  and  $D_t^i$  that depend only on the most recently observed values of the payoff relevant state variables  $X_t$  and  $K_{t-} \equiv (K_{t-}^1, K_{t-}^2)$ , where  $K_{t-}^i \equiv \lim_{s \uparrow t} K_s^i$ . A pure strategy Markov-perfect equilibrium (MPE) of the game is a pair of strategies  $(Q^i, D^i)$ ,  $i = 1, 2$ , such that the value functions (3) are maximized in every state  $(K_{t-}, X_t)$  given the equilibrium strategy of the rival.

It is straightforward to show that any MPE must have quantity choices equal to static Cournot equilibrium output levels. Given our assumption

that demand is sufficiently elastic (implied by  $\gamma > 0$ ) and the absence of marginal costs, all firms produce at full capacity. Hence, any MPE strategy requires  $Q_t^i = Q^i(K_t^i)$ .<sup>8</sup> The instantaneous profit functions

$$\pi_t^i = X_t [Q^1(K_t^1) + Q^2(K_t^2)]^{\gamma-1} Q^i(K_t^i) - F_t^i$$

are thus fully determined by the current capital levels  $K_t^1$  and  $K_t^2$  and the value of the state variable  $X_t$ .

To aid future exposition, it is convenient to define the capital dependent *revenue factors*

$$\begin{aligned} R_{mn}^1 &\equiv [Q^1(\kappa_m) + Q^2(\kappa_n)]^{\gamma-1} Q^1(\kappa_m), \\ R_{mn}^2 &\equiv [Q^1(\kappa_m) + Q^2(\kappa_n)]^{\gamma-1} Q^2(\kappa_n), \end{aligned}$$

where  $m, n \in \{0, 1, 2\}$  index the capital levels of firms 1 and 2, respectively. We can then conveniently write the profit function of each individual firm  $i$  as  $\pi^i(K_t^1 = \kappa_m, K_t^2 = \kappa_n, X_t) = X_t R_{mn}^i - F_t^i$ .

Given the simplification of the instantaneous output choices  $Q_t^i$ , we can henceforth focus attention on the dynamic game of option exercise involving the capital levels  $K_t^i$  and the investment decisions  $D_t^i$ . Any Markov strategy can be summarized by a set of exercise boundaries that for each player  $i$  and each capital state  $K_{t-}$  specify regions of the state variable  $X_t$  at which player  $i$  will change his capital level to a new state. We can use standard techniques of backward induction to derive MPE of the dynamic game.

---

<sup>8</sup>Instantaneous suboptimal actions are ruled out by Markov perfect equilibrium, which requires that all players' strategies must depend only on payoff relevant state variables.

### 3 Rival Growth Options and Risk

This section considers the simplest case of the general model developed in Section 2. Specifically, we assume that one rival is *flexible*, and begins with one option to either expand or contract, while the other rival is *inflexible* and has no ability to change its capital level. This scenario helps us to isolate the two sources of real option risk, own and rival, that can occur in a real options duopoly.

The flexible firm has risk that changes over time only because of its own real option and operating leverage. As in the monopoly case explored in previous literature, the flexible firm has an own-option risk component but no independent source of dynamic industry risk. By contrast, the inflexible firm has no own-option component in its risk loadings, but nonetheless, it is exposed to dynamic risk due to the investment decisions of its rival.

To achieve a specification where one firm is flexible and the other inflexible, we set the capital adjustment costs to

$$\Lambda^1 \equiv \begin{bmatrix} 0 & -\infty & -\infty \\ S & 0 & -I \\ -\infty & -\infty & 0 \end{bmatrix} \quad \Lambda^2 \equiv \begin{bmatrix} 0 & -\infty & -\infty \\ -\infty & 0 & -\infty \\ -\infty & -\infty & 0 \end{bmatrix},$$

where  $S, I > 0$ . Firm 1, the flexible firm, thus begins at capital level  $\kappa_1$  and has a single option to change capacity, either by expanding to  $\kappa_2$  or contracting to  $\kappa_0$ . If it expands, it pays the investment cost  $I$  and if it contracts it receives the salvage value  $S$ . Once firm 1 either expands or contracts, it has no further options to change capacity. Firm 2 begins at

capital level  $\kappa_1$  and has no real options.

We now examine the exercise decision and valuation of the flexible firm.

**Proposition 1:** *The optimal policy of the flexible firm is to expand at  $X_E > 0$  and contract at  $X_C < X_E$ , where  $X_C$  and  $X_E$  solve the pair of nonlinear equations given in the Appendix. The value of the flexible firm prior to option exercise is:*

$$V^1(K_t, X_t) = V_A^1(K_t, X_t) + V_F^1(K_t) + V_O^1(K_t, X_t),$$

where  $V_A^1(K_t, X_t) = R_{11}^1 X_t / \delta$  is the growing perpetuity value of assets in place assuming no future capacity adjustments by either firm,  $V_F^1(K_t) = -f(K_t^1) / r$  is the perpetuity value of fixed operating costs,  $V_O^1(K_t, X_t) = B_1^1 X_t^{\nu_1} + B_2^1 X_t^{\nu_2}$  is the value of growth options,  $B_1^1$  and  $B_2^1$  are positive constants determined by the boundary conditions, and  $\nu_1 > 1$  and  $\nu_2 < 0$  are constants given in the Appendix.

As in standard real option models, (e.g., McDonald and Siegel, 1985, 1986), the flexible firm value consists of assets in place and its own option value. The real option has two components related to the growth opportunity and contraction option respectively, but their values are not independent since the constants  $B_1^1$  and  $B_2^1$  can only be determined by jointly solving the value matching equations at the exercise boundaries. The positivity of the constants  $B_1^1$  and  $B_2^1$  reflects that ownership of these options is value-enhancing, and the positive and negative signs of the roots  $\nu_1$  and  $\nu_2$  reflect that growth

options increase with movements in the underlying asset while the opposite holds for contraction options.

The inflexible firm value consists only of its assets in place, but an externality is imposed by the rival real options.

**Proposition 2.** *The value of the inflexible firm is entirely determined by the value of the assets in place net of the present value of fixed costs:*

$$V^2(K_t, X_t) = V_A^2(K_t, X_t) + V_F^2(K_t) + V_C^2(K_t, X_t),$$

where  $V_A^2(K_t, X_t) = R_{11}^2 X_t / \delta$  is the growing perpetuity value of assets in place assuming no future adjustment to capacity by either firm,  $V_F^2(K_t) = -f(K_t^2) / r$ , is the perpetuity value of fixed operating costs,  $V_C^2(K_t, X_t) = B_1^2 X_t^{\nu_1} + B_2^2 X_t^{\nu_2}$  is the value externality imposed by competitor growth options, and the constants  $B_1^2 \leq 0$ ,  $B_2^2 \geq 0$  are determined by the value matching conditions at the rival exercise boundaries, as described in the Appendix.

The valuation externality imposed by competitor real options has two components related to the rival growth option and contraction option respectively. The negative sign of  $B_1^2$  reflects that rival expansion options reduce value, while  $B_2^2 \geq 0$  follows from the value enhancing effect of competitor contractions. We note from Propositions 1 and 2 that contraction options impact own and rival-firm values with the same sign, whereas expansion options have opposite valuation impacts on a firm and its rivals. These valuation effects have implications for risk.

To determine dynamic loadings on the stochastic discount factor, we

calculate the elasticity of firm value with respect to  $X_t$  as described in the Appendix.

**Proposition 3.** *The dynamic betas for the flexible and inflexible firm are:*

$$\beta^i(K_t, X_t) = 1 + \frac{f_1/r}{V^i(K_t, X_t)} + \left\{ (\nu_1 - 1) \frac{B_1^i X_t^{\nu_1}}{V^i(K_t, X_t)} + (\nu_2 - 1) \frac{B_2^i X_t^{\nu_2}}{V^i(K_t, X_t)} \right\}$$

*prior to option exercise and  $\beta^i(K_t, X_t) = 1 + (f_1/r) / V^i(K_t, X_t)$  afterwards.*

Consistent with the valuation equations, the betas for the flexible and inflexible firms consist of three parts. By assumption the revenue beta is equal to 1. The second component is operating leverage, which always increases risk, and the final term for both firms arises from the flexible firm's real options. We note that although the structure of beta for both firms is similar, the economic interpretation is very different. The flexible firm's risk depends only on its own firm-specific decisions, whereas the inflexible firm has no decisions to make and its risk is determined entirely by industry effects.

Examining the flexible firm first, we note that since  $\nu_1 > 1$  and  $B_1^1 \geq 0$ , its own risk rises due to its own option to expand. On the other hand, since  $\nu_2 < 0$  and  $B_2^1 \geq 0$ , the option to contract reduces risk. The inflexible firm dynamic loadings on the stochastic discount factor are determined by  $B_1^2 \leq 0$  and  $B_2^2 \geq 0$ , implying that its risk decreases due to both the competitor growth option and the competitor expansion option. This simple example illustrates two important points, which we now discuss in more detail.

First, rival real options reduce risk. Intuitively, a competitor's invest-

ment decisions act as a natural hedge against variations in the exogenous state variable. Good news about demand going up will be partially offset by the bad news that the competitor is closer to expanding. Figure 1 gives a graphical presentation of this hedging argument. Before the flexible firm exercises her option, industry demand is indicated by the downward sloping curve  $D$  and the industry supplies output at the full-capacity level  $Q_1$ . Consider now an increase in demand to the level  $D'$  that induces the flexible firm to exercise her growth option. The corresponding increase in industry supply causes prices to increase less than to the level  $P^*$  corresponding to the old supply curve. Prices rise more moderately to  $P_2$  instead of  $P^*$ , and the dampening in profits caused by the increase in industry supply after a positive demand shock corresponds to the natural hedging effect caused by rival real options.

The second important implication of the simple example developed in this section is that expansion options have an oppositely signed impact on own-firm and rival risk, while contraction options affect both firms' risk in the same direction. These risk implications follow from the valuation impacts of own and rival real options discussed previously. Contraction options of both one's own firm and rivals create a hedge against adverse moves in underlying fundamentals. By contrast, own-firm expansion opportunities amplify risk, whereas rival expansion opportunities mitigate the potential for upside gain.

Figure 2 shows the own and rival risk effects discussed above. For simplicity, we assume the inflexible firm has no operating leverage. In the figure,  $X_C$  is the critical level of demand at which the flexible firm shrinks and  $X_E$

is critical level at which the flexible firm expands. The diagram illustrates that rival real options reduce risk, and that real options can cause own and competitor risks to move together or in opposite directions. As demand increases and the growth option becomes more important, the flexible firm's risk increases while the inflexible firm's risk decreases. By contrast, when demand decreases and the contraction option is more valuable, own and rival firm risk tend to move together. The next section investigates the robustness of these results when both firms possess growth options and exercise is strategic.

## 4 Dynamic Risk in Asymmetric Industry Equilibrium

We now permit both firms to have real options to expand or contract capacity, and consider the corresponding equilibrium play. We obtain analytical solutions for firm risk and required return in two cases: 1) when both firms have expansion options, and 2) when both firms have contraction options.

### 4.1 Equilibrium Exercise of Expansion Options

To analyze equilibrium in the case where both firms have only a single growth option, we set the capital adjustment costs to

$$\Lambda^1 \equiv \begin{bmatrix} 0 & -\infty & -\infty \\ -\infty & 0 & -I \\ -\infty & -\infty & 0 \end{bmatrix} \quad \Lambda^2 \equiv \begin{bmatrix} 0 & -\infty & -\infty \\ -\infty & 0 & -\rho I \\ -\infty & -\infty & 0 \end{bmatrix},$$

where  $\rho \geq 1$  so that the expansion costs of firm 1 are lower than those of firm 2.

We show below that in all Markov-perfect equilibria the low-cost firm invests at least as soon as the high-cost firm. Given this simplification, industry structure can be one of three potential phases: a *juvenile* industry where neither firm has exercised its growth option, an *adolescent* industry where the “leader” has exercised and the “follower” has not, and a *mature* industry where both firms have expanded. Figure 3 depicts the different industry stages.

We divide all possible equilibria into two primary classifications, “simultaneous” and “leader-follower.” By this categorization, a leader-follower equilibrium is defined simply by the absence of simultaneous exercise. All three industry stages occur for a finite period of time with probability one in a leader-follower equilibrium, whereas in a simultaneous equilibrium the industry structure jumps immediately from juvenile to mature.

Following Pawlina and Kort (2006),<sup>9</sup> determining payoffs under different strategies proceeds by backward induction, and allows determination of the type of equilibrium. For example, assuming firm 1 as the leader and firm 2 as the follower, we first calculate the optimal exercise of firm 2 and then the optimal exercise of firm 1. We similarly calculate value functions and triggers with firm 2 as the leader and firm 1 as the follower. Finally, we calculate value functions and triggers where both firms exercise simultaneously.

To simplify discussion, in the remainder of this section we assume that the initial demand state  $X_0$  is strictly less than the leader trigger level of

---

<sup>9</sup>See also Smets (1991) and Dixit and Pindyck (1994).

firm 1, which ensures that the juvenile industry state occurs for a finite period of time in equilibrium. We then summarize the types of equilibrium that may occur.

**Proposition 4.** *The MPE of the expansion game are characterized by:*

1. *Simultaneous Equilibrium: There exists a value  $\rho^{**}(\gamma, q_1, q_2, f_1, f_2, v_1) > 0$  such that for all  $1 \leq \rho < \rho^{**}$  the unique MPE involves simultaneous exercise with a trigger that maximizes the low-cost firm's value. There exist no other simultaneous investment MPE. Hence if  $\rho^{**} < 1$ , no simultaneous exercise equilibria exist.*
2. *Non-Preemptive Leader-Follower Equilibrium: There exists a value  $\rho^*(\gamma, q_1, q_2, f_1, f_2, v_1) > 1$  such that for all  $\rho > \max[\rho^*, \rho^{**}]$ , the unique MPE results in the high-cost firm acting as the follower with trigger  $X_F^2$  and the low-cost firm acting as the leader with trigger  $X_{LN}^1 < X_F^2$ , where the triggers given in the Appendix are identical to those obtained if the roles of leader and follower were predetermined prior to the beginning of the game, and the follower could not threaten to preempt the leader's investment.*
3. *Preemptive Leader-Follower Equilibrium: For  $\rho$  satisfying  $\rho^* \geq \rho \geq \max[\rho^{**}, 1]$ , the unique MPE results in the high-cost firm acting as the follower with trigger  $X_F^2$  and the low-cost firm acting as the leader with trigger  $X_{LP}^1 < X_F^2$ , where the trigger of the leader  $X_{LP}^1 \leq X_{LN}^1$  is determined as the indifference point of the high-cost firm between acting as a leader or a follower. Hence the threat of the high-cost firm*

to preemptively expand causes the low-cost firm to itself preemptively invest just at the instant when the high-cost firm's preemption threat becomes credible. The leader's expansion deters growth of the follower in the region between  $X_{LP}^1$  and  $X_F^2$ .

4. *Random Leader-Follower Equilibrium:* If  $\rho = 1$  and  $\rho^{**} < 1$ , no pure strategy MPE is possible. To obtain a mixed strategy MPE requires expanding the strategy space as discussed in Fudenberg and Tirole (1985) and Huisman and Kort (1999).<sup>10</sup> In the mixed strategy equilibrium the leader is randomly chosen at instant  $X_{LP}^1 = X_{LP}^2$ , and the other firm becomes the follower exercising at  $X_F^1 = X_F^2$ .

Equilibrium play in the expansion game can thus be categorized by the regions of the parameter space in which each equilibrium holds. To illustrate the proposition, we fix the parameter values  $\sigma = 0.2, q_1 = 2, q_2 = 10, f_1 = f_2 = 0, r = 0.05, \delta = 0.03$  and  $I = 500$ , and diagram in Figure 4 the equilibrium regions in the two-dimensional space of  $\gamma$ , related to the demand elasticity, and the relative cost difference  $\rho$ . For high levels of demand elasticity (low  $\gamma$ ), the simultaneous equilibrium can be supported even when expansion cost asymmetries are large. By contrast when the demand elasticity is low (high  $\gamma$ ), arbitrarily small positive cost asymmetries imply that one of the pure strategy leader-follower equilibria must hold.<sup>11</sup>

<sup>10</sup>See also Thijssen, Huisman, and Kort (2002) and Boyer, Lasserre, Mariotti, and Moreaux (2001).

<sup>11</sup>An interesting comparative static that does not appear in Figure 4 is that as  $q_2$  increases (corresponding to an increase in the ratio of growth options to assets-in-place), the region corresponding to simultaneous investment shrinks.

The link between demand elasticity and the existence of the simultaneous exercise equilibrium relates to the impact of investment on the profits generated by assets in place. When demand elasticity is low, expanding output has a small negative effect on asset-in-place value, and the benefit of waiting for simultaneous investment relative to acting as a leader is not as large. By contrast, when the demand elasticity is very high the value of waiting to invest simultaneously can be everywhere higher than the value of acting as a leader, and simultaneous investment can be supported.

Valuation in the pure-strategy leader-follower equilibria can be conveniently summarized.

**Proposition 5.** *In any pure-strategy leader-follower equilibrium, the leader's value function  $V^1(K_t, X_t)$  is given by*

$$\left\{ \begin{array}{ll} \frac{R_{11}^1 X_t}{\delta} - \frac{f_1}{r} + \left[ \frac{(R_{21}^1 - R_{11}^1) X_L^1}{\delta} - \frac{(f_2 - f_1 + rI)}{r} \right] \left( \frac{X_t}{X_L^1} \right)^{\nu_1} & X_t < X_L^1, \\ + \frac{X_F^2}{\delta} [R_{22}^1 - R_{21}^1] \left( \frac{X_t}{X_F^2} \right)^{\nu_1} & \\ \frac{R_{21}^1 X_t}{\delta} - \frac{f_2}{r} + \frac{X_F^2}{\delta} [R_{22}^1 - R_{21}^1] \left( \frac{X_t}{X_F^2} \right)^{\nu_1} & X_L^1 \leq X_t \leq X_F^2, \\ \frac{R_{22}^1 X_t}{\delta} - \frac{f_2}{r} & X_t > X_F^2, \end{array} \right.$$

and the follower's value function  $V^2(K_t, X_t)$  is equal to

$$\left\{ \begin{array}{ll} \frac{R_{11}^2 X_t}{\delta} - \frac{f_1}{r} + \frac{f_2 - f_1 + r\rho I}{r(\nu_1 - 1)} \left( \frac{X_t}{X_F^2} \right)^{\nu_1} + \frac{X_L^1}{\delta} [R_{21}^2 - R_{11}^2] \left( \frac{X_t}{X_L^1} \right)^{\nu_1} & X_t \leq X_L^1, \\ \frac{R_{21}^2 X_t}{\delta} - \frac{f_1}{r} + \frac{f_2 - f_1 + r\rho I}{r(\nu_1 - 1)} \left( \frac{X_t}{X_F^2} \right)^{\nu_1} & X_L^1 \leq X_t \leq X_F^2, \\ \frac{R_{22}^2 X_t}{\delta} - \frac{f_2}{r} & X_t > X_F^2. \end{array} \right.$$

where the optimal leader trigger is  $X_L^1 \equiv X_{LN}^1$  for a non-preemptive equilib-

rium and  $X_L^1 \equiv X_{LP}^1$  in a preemptive equilibrium.

Both firm values are composed of the growing perpetuity value of the assets in place assuming constant industry structure, the perpetuity value of the fixed costs, the own-firm option value, and the externality imposed by the rival option.

Using the leader's non-preemptive trigger  $X_{LN}^1 = (f_2 - f_1 + rI)\delta\nu_1 / [(R_{21}^1 - R_{11}^1)r(\nu_1 - 1)]$ , an alternative decomposition of the leader value function in a juvenile industry can be derived:

$$\begin{aligned}
 V^1(K_t, X_t) = & \underbrace{\frac{R_{11}^1}{\delta} X_t - \frac{f_1}{r}}_{\text{assets in place}} + \underbrace{\left[ X_L^1 - \left( 1 - \frac{1}{\nu_1} \right) X_{LN}^1 \right] \frac{[R_{21}^1 - R_{11}^1]}{\delta} \left( \frac{X_t}{X_L^1} \right)^{\nu_1}}_{\text{own growth option}} \\
 & + \underbrace{\frac{X_F^2}{\delta} [R_{22}^1 - R_{21}^1] \left( \frac{X_t}{X_F^2} \right)^{\nu_1}}_{\text{rival value adjustment}}, \tag{4}
 \end{aligned}$$

where as before  $X_L^1 \equiv X_{LN}^1$  applies for a non-preemptive equilibrium and  $X_L^1 \equiv X_{LP}^1$  for a preemptive one. The rival value adjustment is always negative, consistent with the price-reducing effect of competitor expansion discussed in Section 3. The leader's own growth option value is proportional to a weighted sum of the two triggers  $X_L^1$  and  $X_{LN}^1$ . In a non-preemptive equilibrium  $X_L^1 = X_{LN}^1$  it is straightforward to observe that the leader's own growth option value must be positive since  $\nu_1 > 1$ . In a preemptive equilibrium, equation (4) interestingly shows that both the preemptive and non-preemptive triggers enter into the valuation equation, and the relative sizes of the two triggers as well as  $\nu_1$  determine the sign of the own growth

option value. For most parameters, the own growth option value is positive, but for example if the risk-free rate and hence  $v_1$  are very large then the leader's own growth option value can in fact be negative.<sup>12</sup>

The follower's value in a juvenile industry similarly can be rewritten:

$$\begin{aligned}
 V^2(K_t, X_t) = & \underbrace{\frac{R_{11}^2}{\delta} X_t - \frac{f_1}{r}}_{\text{assets in place}} + \underbrace{\frac{X_F^2}{\delta \nu_1} [R_{22}^2 - R_{21}^2] \left( \frac{X_t}{X_F^2} \right)^{\nu_1}}_{\text{own growth option}} \\
 & + \underbrace{\frac{X_L^1}{\delta} [R_{21}^2 - R_{11}^2] \left( \frac{X_t}{X_L^1} \right)^{\nu_1}}_{\text{rival value adjustment}}.
 \end{aligned}$$

The follower's own-option effect is always positive and the rival value-adjustment is negative.

The valuation equations in a simultaneous-exercise equilibrium have a different appearance.

**Proposition 6.** *In case of simultaneous exercise the value function of each firm  $V^i(K_t, X_t)$  is given by*

$$\begin{cases} \frac{R_{11}^i X_t}{\delta} - \frac{f_1}{r} + \frac{f_2 - f_1 + r I^i}{r(\nu_1 - 1)} \left( \frac{X_t}{X_S^i} \right)^{\nu_1} & X_t \leq X_S^i, \\ \frac{R_{22}^i X_t}{\delta} - \frac{f_2}{r} & X_t > X_S^i, \end{cases}$$

with the expansion trigger  $X_S^i = \nu_1 \delta (f_2 - f_1 + r I^i) / [(\nu_1 - 1)r (R_{22}^i - R_{11}^i)]$ .

Firm value therefore appears to contain only the assets in place and an

---

<sup>12</sup>One set of parameters for which the leader own growth option value is negative is  $\gamma = 0.5$ ,  $\sigma = 0.1$ ,  $q_1 = 2$ ,  $q_2 = 10$ ,  $f_1 = f_2 = 0$ ,  $r_f = 1.0$ ,  $\delta = 0.88$ ,  $I = 100$ .

option component:

$$V^i(K_t, X_t) = \underbrace{\frac{R_{11}^i}{\delta} X_t - \frac{f_1}{r}}_{\text{assets in place}} + \underbrace{\frac{X_S^i}{\delta \nu_1} [R_{22}^i - R_{11}^i] \left( \frac{X_t}{X_S^i} \right)^{\nu_1}}_{\text{growth option}}. \quad (5)$$

Hence, the distinguishing feature of the simultaneous exercise equilibrium is that the rival firm value adjustment is not apparent.

Of course, both own-firm and rival effects are implicitly embedded within the growth option component of (5), but there is not a unique decomposition of the change in profits  $R_{22}^i - R_{11}^i$ . For example, one possible decomposition for firm 1 is to designate  $R_{22}^1 - R_{12}^1$  as the own growth option component and  $R_{12}^1 - R_{11}^1$  as the rival effect. On the other hand, it is equally sensible to view  $R_{22}^1 - R_{21}^1$  as the competitor effect and  $R_{21}^1 - R_{11}^1$  as the own effect. Thus, due to simultaneous exercise the own and rival effects are not separately identified. However, we do note that in both possible decompositions the own effect is positive and the rival effect is negative.

To derive risk implications for all three different types of equilibria, we use similar notation as previously and write for  $i = 1, 2$ ,

$$V^i(K_t, X_t) = V_A^i(K_t, X_t) + V_F^i(K_t) + V_O^i(K_t, X_t) + V_C^i(K_t, X_t),$$

where  $V_O^i(K_t, X_t)$  is the own-option component of value, and  $V_C^i(K_t, X_t)$  is the rival-option component of value.<sup>13</sup> We then show

---

<sup>13</sup> We acknowledge that the own and rival components can interact, particularly in the pre-emption equilibrium where the follower real option directly influences the leader trigger. However, conditional on the leader and follower triggers the decomposition into own and rival components is natural.

**Proposition 7.** *In all pure strategy equilibria, systematic firm risks for the follower and the leader are given by*

$$\beta^i(K_t, X_t) = 1 + \frac{V_O^i(K_t, X_t) + V_C^i(K_t, X_t)}{V^i(K_t, X_t)}(\nu_1 - 1) + \frac{V_F^i(K_t)}{V^i(K_t, X_t)}, \quad (6)$$

for all industry states  $K_t$ .

Systematic firm risk in a growing oligopolistic industry is thus driven by a firm's operating leverage, its own growth options, and the risk reducing effects of rival growth options.

We emphasize several important points regarding Proposition 7. First, the own growth option and rival growth option components of value enter additively into the second term in (6). Hence, the risk effects of own and rival growth options are identical when normalized by dollar values, which provides a remarkable simplification. Second, since  $\nu_1 > 1$  and  $V_C^i(K_t, X_t) < 0$ , rival growth options always reduce risk, independent of whether the equilibrium is simultaneous, pre-emptive, or non-preemptive. Third, when  $V_O^i(K_t, X_t) > 0$ , which always holds for non-preemptive equilibria, own-firm expansion options increase risk.

Perhaps most importantly, only in the simultaneous equilibrium can we uniquely sign the sum  $V_O^i(K_t, X_t) + V_C^i(K_t, X_t)$  at all points in the state space. In particular, in a simultaneous exercise equilibrium this sum is guaranteed to be positive, and hence the cumulative effect of growth options is always to increase risk, consistent with the results in Aguerrevere (2009). By contrast, in a leader-follower equilibrium the cumulative effect of industry growth options  $V_O^i(K_t, X_t) + V_C^i(K_t, X_t)$  can generally not be

uniquely signed, implying that growth options will alternately increase or decrease risk for a given firm at different points in the state space.

Figure 5 displays the evolution of risk in a growing industry for all three different types of equilibria. In the four panels of the figure, we hold all parameters constant except the expansion cost asymmetry which is set to 2.0 in Panel A, 1.3 in Panel B, 1.1 in Panel C, and 1.0 in Panel D. As a consequence, in Panels A and B the equilibrium type is non-preemptive, in Panel C the equilibrium is preemptive, and in panel D the equilibrium is simultaneous. The equilibrium types are consistent with Figure 2 where the demand elasticity is set to the intermediate value  $\gamma = 0.5$ .

Moving from Panel A to B by decreasing the adjustment cost asymmetry  $\rho$ , the follower trigger moves forward closer to the leader trigger, but has no strategic impact on the leader decision of when to exercise, consistent with the nature of the non-preemptive equilibrium. However, in Panel C, the follower trigger moves close enough to the leader trigger that the follower would have an incentive to strategically preempt the leader prior to its non-preemptive trigger. As a consequence, the leader must itself preempt the preemptive investment of the follower, by moving forward its trigger to  $X_{LP}^1$ . Finally, in Panel D the firms are sufficiently symmetric and the option value of waiting relative to acting as a leader sufficiently large that a simultaneous exercise equilibrium can be sustained.

The risk dynamics of the two firms in the leader-follower equilibria in Panels A-C differ markedly from the simultaneous exercise equilibrium in Panel D. For a leader-follower equilibrium, prior to the leader's exercise the leader's risk increases more steeply than the follower. Immediately upon

the exercise of the leader growth option, the leader risk drops discretely and the follower risk jumps upwards, reversing the risk-ordering of the two firms. The two firms' risk loadings continue to move apart until the follower growth option is exercised, and no growth options remain. By contrast, under simultaneous exercise in Panel D, the risk of both firms increases equally and always is above one until the exercise trigger and then drops to the level of the cash flow beta.

The dynamics of risk in a leader-follower equilibrium therefore differ dramatically from the simultaneous exercise case. Proposition 4 states that the region of existence of the simultaneous equilibrium can be small depending on parameter values, and in many cases the simultaneous exercise equilibrium does not exist even for perfectly symmetric firms. These results imply that leader-follower equilibria are important and merit independent study. Our theoretical investigation shows that growth options have opposite effects on own firm and rival risk. Hence, the common practice of proxying for a firm's risk using industry peer betas may not be appropriate when growth options are an important component of firm values, and this theoretical implication can be tested in future empirical work.

## 4.2 Equilibrium Exercise of Contraction Options

We now assume that each firm has a single contraction option. Capital adjustment costs are specified by

$$\Lambda^1 \equiv \begin{bmatrix} 0 & -\infty & -\infty \\ S & 0 & -\infty \\ -\infty & -\infty & 0 \end{bmatrix} \quad \Lambda^2 \equiv \begin{bmatrix} 0 & -\infty & -\infty \\ \rho S & 0 & -\infty \\ -\infty & -\infty & 0 \end{bmatrix},$$

where  $0 < \rho \leq 1$  so that firm 1 has the high and firm 2 the low salvage value. This implies that firm 1 has an incentive to contract earlier. Our interest again lies in equilibrium play of the two rivals, and we focus on pure strategy equilibria only. In contrast to the case of expansion options, leader-follower exercise is the unique equilibrium, following Murto (2004).

**Proposition 8.** *For every  $0 < \rho < 1$  there exists a unique MPE of the contraction game in which the high salvage value firm acts as the leader with trigger  $X_C^1$  and the low salvage value firm acts as the follower with trigger  $X_C^2 < X_C^1$ . If  $\rho = 1$ , there exist two pure strategy MPE, one in which firm 1 acts as the leader and firm 2 as the follower, and in the other firm 2 acts as the leader and firm 1 as the follower. No equilibrium exists with positive probability of simultaneous contraction.*

We note that preemption does not play a role in contractions because any rival reduction in output increases rather than reduces firm value. Hence, for symmetric firms the follower value exceeds the leader value.

We again assume both firms initially have capacity  $\kappa_1$ . The leader con-

tracts first at the demand trigger  $X_C^1$ , and the follower contracts at the trigger  $X_C^2 < X_C^1$ . We then show:

**Proposition 9.** *The leader's value function  $V^1(K_t, X_t)$  is equal to*

$$\begin{cases} \frac{R_{11}^1 X_t}{\delta} - \frac{f_1}{r} + \frac{f_1 - f_0 + rS}{r(1-\nu_2)} \left(\frac{X_t}{X_C^1}\right)^{\nu_2} + \frac{X_C^2}{\delta} [R_{00}^1 - R_{01}^1] \left(\frac{X_t}{X_C^2}\right)^{\nu_2} & X_t > X_C^1, \\ \frac{R_{01}^1 X_t}{\delta} - \frac{f_0}{r} + \frac{X_C^2}{\delta} [R_{00}^1 - R_{01}^1] \left(\frac{X_t}{X_C^2}\right)^{\nu_2} & X_C^2 \leq X_t \leq X_C^1, \\ \frac{R_{00}^1 X_t}{\delta} - \frac{f_0}{r} & X_t < X_C^2, \end{cases}$$

and the followers value  $V^2(K_t, X_t)$  is

$$\begin{cases} \frac{R_{11}^2 X_t}{\delta} - \frac{f_1}{r} + \frac{f_1 - f_0 + r\rho S}{r(1-\nu_2)} \left(\frac{X_t}{X_C^2}\right)^{\nu_2} + \frac{X_C^1}{\delta} [R_{01}^2 - R_{11}^2] \left(\frac{X_t}{X_C^1}\right)^{\nu_2} & X_t \geq X_C^1, \\ \frac{R_{01}^2 X_t}{\delta} - \frac{f_1}{r} + \frac{f_1 - f_0 + r\rho S}{r(1-\nu_2)} \left(\frac{X_t}{X_C^2}\right)^{\nu_2} & X_C^2 \leq X_t \leq X_C^1, \\ \frac{R_{00}^2 X_t}{\delta} - \frac{f_0}{r} & X_t < X_C^2. \end{cases}$$

with the contraction triggers  $X_C^1 > X_C^2 > 0$  given in the Appendix.

Rewriting the contraction triggers and substituting into the value functions gives

$$\begin{aligned} V^1(K_t, X_t) = & \underbrace{\frac{R_{11}^1 X_t}{\delta} - \frac{f_1}{r}}_{\text{assets in place}} + \underbrace{\frac{X_C^1}{\delta \nu_2} [R_{01}^1 - R_{11}^1] \left(\frac{X_t}{X_C^1}\right)^{\nu_2}}_{\text{contraction option}} \\ & + \underbrace{\frac{X_C^2}{\delta} [R_{00}^1 - R_{01}^1] \left(\frac{X_t}{X_C^2}\right)^{\nu_2}}_{\text{value adjustment}}. \end{aligned}$$

and

$$\begin{aligned}
V^2(K_t, X_t) = & \underbrace{\frac{R_{11}^2}{\delta} X_t - \frac{f_1}{r}}_{\text{assets in place}} + \underbrace{\frac{X_C^2}{\delta \nu_2} [R_{00}^2 - R_{01}^2]}_{\text{contraction option}} \left( \frac{X_t}{X_C^2} \right)^{\nu_2} \\
& + \underbrace{\frac{X_C^1}{\delta} [R_{01}^2 - R_{11}^2]}_{\text{value adjustment}} \left( \frac{X_t}{X_C^1} \right)^{\nu_2}.
\end{aligned}$$

The value functions are composed of the growing perpetuity value of assets net of fixed costs assuming a constant industry structure, the perpetuity value of fixed costs, and the own-firm and rival option effects. The own-firm contraction option corresponds to a put and has positive value, consistent with the product of  $\nu_2 < 0$  and  $[R_{11}^1 - R_{01}^1] < 0$ . The rival value adjustment also has a positive value, consistent with the increased market price induced by lower industry output.

As in the expansion case, the value functions can be written as:

$$V^i(K_t, X_t) = V_A^i(K_t, X_t) + V_F^i(K_t, X_t) + V_O^i(K_t, X_t) + V_C^i(K_t, X_t).$$

In contrast to expansion options, the rival effect for downsizing is positive  $V_C^i(K_t, X_t) > 0$ .

The risk dynamics of the two firms follows from the valuation equations.

**Proposition 10.** *Systematic firm risks for both firms are*

$$\beta^i(t) = 1 + \frac{V_O^i(K_t, X_t) + V_C^i(K_t, X_t)}{V^i(K_t, X_t)} (\nu_2 - 1) + \frac{f_k/r}{V^i(K_t, X_t)},$$

for all industry states  $K_t$ , where  $\nu_2 < 0$  and  $V_O^i(K_t, X_t), V_C^i(K_t, X_t) > 0$ .

As in the case of expansion options, the own-firm and rival values of contractions appear additively in the numerator of the second term, again implying that own and competitor contraction options have the same risk implications when normalized by dollar values. In contrast to the case of expansion options the signs of  $V_O^i$  and  $V_C^i$  are always positive, which combined with  $\nu_2 < 0$  implies that contraction options, whether own or rival, always reduce risk.

Figure 6 summarizes the risk dynamics in equilibrium for contraction options. In Panel A the degree of salvage value asymmetry is large with  $\rho = 0.1$ , and in the remaining three panels  $\rho$  progressively increases until reaching  $\rho = 0.99999$ . As  $\rho$  increases and the follower salvage value increases, its contraction trigger moves closer to the leader's. However, unlike in the expansion case, the increase in the follower trigger has no strategic impact on the leader's exercise, which always occurs at the same level of demand. We also note that even in the case where the salvage value is almost one, the difference in the leader and follower triggers is discrete. This is because the exit of the leader raises the incentives of the follower to delay, so that the two triggers cannot occur arbitrarily close together.

The risk dynamics of the two firms in the contraction equilibrium differ, with each firm's risk dropping faster prior to its own capacity reduction. However, consistent with Proposition 10, both contraction options reduce risk for both firms, and the firms' risks always move in the same direction.

## 5 Conclusion

We study risk dynamics in a duopoly where firms possess real options to expand or contract capacity, with adjustment costs that potentially differ across firms. Prior research derives required returns in a variety of product market settings: monopoly, where own-firm real options and operating leverage impact returns (Carlson, Fisher and Giammarino, 2004, 2006; Cooper 2006); perfect competition with identical firms, where only industry effects are present (Kogan, 2004); perfect competition with heterogeneous firms, where option values are zero and differences in cost structure drive expected returns (Zhang, 2005); and oligopoly, where both own and rival real options exist but their distinct impacts are not apparent due to simultaneous exercise (Aguerrevere, 2009). In the duopoly setting that we analyze, a variety of leader-follower equilibria exist in which a firm and its rival may exercise growth and contraction opportunities at separate times, allowing identification of the distinct risk impacts of own and competitor growth and contraction options. Non-simultaneous exercise can be the unique equilibrium outcome even when exogenous asymmetries are arbitrarily small or zero, and the risk dynamics that emerge in leader-follower equilibria differ substantially from the simultaneous trigger benchmarks that have been more commonly studied in the literature.

We find that a competitor's options to adjust capacity, whether expansion or contraction, reduce own-firm risk through a simple hedging channel. Intuitively, product market improvements increase the probability of near-term rival expansion, which provides an offsetting decrease in own-firm

value. Conversely, negative demand shocks induce competitor contraction, reducing the decline in own-firm value. As a consequence of the risk-reducing effect of competitor real options, own and rival risk tend to move together in contractions, but in opposite directions during expansions.

Financial analysts commonly estimate the required return of a product or corporation using not only the historical risk of the firm, but also its industry rivals, as recommended by standard corporate finance textbooks. Our results suggest that using industry peer betas to proxy for own-firm risk may work well in certain environments, but not in others, in particular where growth options are an important component of firm value. Our study thus highlights the importance of rival real options as an independent source of firm risk dynamics, and provides new theoretical predictions that can be tested in future empirical work.

## Appendix

### A Proofs

#### A.1 Proof of Proposition One

To derive the value of the flexible firm we assume that there exist two traded assets that can be used to hedge industry demand uncertainty. Let  $B_t$  denote the price of a riskless bond with dynamics  $dB_t = rB_t dt$  where  $r > 0$  is the constant riskless rate of interest, and let  $S_t$  be the price of a risky asset. The price dynamics of the risky asset is given by

$$dS_t = \mu S_t dt + \sigma S_t dW_t.$$

The risky asset  $S_t$  and the industry demand shock  $X_t$  are perfectly correlated. Hence, we can use the securities  $B_t$  and  $S_t$ , to construct a portfolio of the bond and the asset  $S_t$  that perfectly replicates the industry shocks  $X_t$  and derive its risk neutral measure. Demand dynamics under risk neutral measure are given by

$$dX_t = (r - \delta)X_t dt + \sigma X_t d\hat{W}_t, \quad (7)$$

where  $r > \delta \equiv \mu - g > 0$ . All the valuations in this paper are based on the risk neutral dynamics (7).

The continuation value of the flexible firm satisfies the valuation equation

$$\frac{1}{2}\sigma^2 X^2 V_{XX}^1 + (r - \delta)XV_X^1 - rV^1 + XR_{11}^1 - f_1 = 0. \quad (8)$$

with the boundary conditions

$$\begin{aligned}
V^1(X_E) &= \frac{R_{21}^1 X_E}{\delta} - I - \frac{f_2}{r} \\
V^1(X_C) &= \frac{R_{01}^1 X_C}{\delta} + S - \frac{f_0}{r} \\
V_X^1(X_E) &= \frac{R_{21}^1}{\delta} \\
V_X^1(X_C) &= \frac{R_{01}^1}{\delta}.
\end{aligned}$$

The first two equations are the value matching conditions and specify that the option value at the critical boundaries are exactly equal to the present value of the incremental revenues net of adjustment costs. The last two equations are the smooth pasting conditions which are necessary for value maximization. This system of equations has no convenient analytical solution for  $X_E$  and  $X_C$  due to its nonlinearity.

The solution to equation (8) is given by

$$V^1(K_t, X_t) = \frac{R_{11}^1 X_t}{\delta} - \frac{f_1}{r} + B_1^1 X_t^{\nu_1} + B_2^1 X_t^{\nu_2},$$

where  $B_1^1$  and  $B_2^1$  solve

$$\begin{aligned}
(1 - \nu_1)B_1^1 X_E^{\nu_1} + (1 - \nu_2)B_2^1 X_E^{\nu_2} &= -I - \frac{f_2 - f_1}{r}, \\
(1 - \nu_1)B_1^1 X_C^{\nu_1} + (1 - \nu_2)B_2^1 X_C^{\nu_2} &= S - \frac{f_0 - f_1}{r}.
\end{aligned} \tag{9}$$

The constants  $\nu_1 > 1$  and  $\nu_2 < 0$  are the positive and negative roots to the

characteristic equation

$$\frac{1}{2}\sigma^2\nu(\nu - 1) + (r - \delta)\nu - r = 0,$$

and satisfy

$$\nu_{1,2} = \frac{1}{2} - \frac{r - \delta}{\sigma^2} \pm \sqrt{\left(\frac{1}{2} - \frac{r - \delta}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}}.$$

For given positive values of  $X_E$  and  $X_C$  the solution of (9) satisfies  $B_1^1, B_2^1 >$

0. The values  $X_E < X_C$  can be derived numerically.

## A.2 Proof of Proposition Two

The continuation value of the inflexible firm satisfies the valuation equation

$$\frac{1}{2}\sigma^2 X^2 V_{XX}^2 + (r - \delta)XV_X^2 - rV^2 + XR_{11}^2 - f_1 = 0, \quad (10)$$

with two boundary conditions

$$\begin{aligned} V^2(X_E) &= \frac{R_{21}^2 X_E}{\delta} - \frac{f_1}{r} \\ V^2(X_C) &= \frac{R_{01}^2 X_C}{\delta} - \frac{f_1}{r}, \end{aligned}$$

for given trigger levels  $X_E$  and  $X_C$ . The boundary conditions are two value matching conditions. Solving the valuation equation using the value matching conditions results in the value of the flexible firm given by

$$V^2(K_t, X_t) = \frac{R_{11}^2 X_t}{\delta} - \frac{f_1}{r} + B_1^2 X_t^{\nu_1} + B_2^2 X_t^{\nu_2},$$

where  $B_1^2$  and  $B_2^2$  are the solutions to the equations

$$\begin{aligned} B_1^2 X_E^{\nu_1} + B_2^2 X_E^{\nu_2} &= \frac{R_{21}^2 - R_{11}^2}{\delta} X_E - \frac{f_2 - f_1}{r} \\ B_1^2 X_C^{\nu_1} + B_2^2 X_C^{\nu_2} &= \frac{R_{01}^2 - R_{11}^2}{\delta} X_C - \frac{f_0 - f_1}{r}. \end{aligned}$$

For this equation system it is easy to show that  $B_2^1 < 0$  and  $B_2^2 > 0$ .

### A.3 Proof of Proposition Three

Following the arguments in Carlson, Fisher, and Giammarino (2004) betas are given by

$$\beta^i(K_t, X_t) = \frac{\partial V^i(K, X)}{\partial X} \frac{X}{V(K, X)}.$$

Taking partial derivatives and substituting firm values from Propositions 1 and 2 into this expression implies the result.

### A.4 Proof of Proposition Four

We prove existence of the non-preemptive, preemptive, and simultaneous move equilibria and provide conditions when each holds. We also characterize a region for which only a mixed strategy equilibrium is possible. The arguments build on Pawlina and Kort (2006)<sup>14</sup> by permitting operating leverage and accommodating the case where  $\rho = 1$  so that firms 1 and 2 are identical ex ante. We also provide valuation equations for all industry stages juvenile, adolescent, and mature.

---

<sup>14</sup>See also the paper by Mason and Weeds (2008) in which non-preemptive, preemptive and simultaneous-move equilibria in a simple real option game are discussed.

The structure of the proof is to first provide the value functions of firm 1 and 2 under different strategies:

- i) Nonpreemptive leader-follower: firm 1 expands before firm 2 at a demand level that satisfies firm 1's smooth pasting condition.
- ii) Preemptive leader-follower: firm 1 expands before firm 2 but at a demand level forced by firm 2's preemption threat, hence firm 1's exercise not satisfying it's own smooth pasting condition.
- iii) Simultaneous move: both firms expand simultaneously.
- iv) Off-equilibrium: firm 2 leads.

Based on the value functions derived in Part 1, we consider the conditions required to support a simultaneous move equilibrium in Part 2, a preemptive equilibrium in Part 3, and a random move equilibrium in Part 4.

**Part 1: Value function calculations**

*i) Non-preemptive leader-follower:* In this case the value function of firm 1 satisfies

$$\frac{1}{2}\sigma^2 X^2 V_{XX}^1 + (r - \delta)XV_X^1 - rV^1 + XR_{11}^1 - f_1 = 0,$$

with the boundary conditions

$$\begin{aligned} V^1(X_L^1) &= \frac{R_{21}^1 X_L^1}{\delta} - \frac{f_1}{r} - I + B(X_L^1)^{\nu_1} \\ V_X^1(X_L^1) &= \frac{R_{21}^1}{\delta} + B\nu_1 (X_L^1)^{\nu_1-1} \\ V^1(X_F^2) &= \frac{R_{22}^1 X_F^2}{\delta} - \frac{f_2}{r} \end{aligned} \tag{11}$$

where  $X_L^1$  is the exercise trigger optimally set by firm 1,  $X_F^2 > X_L^1$  is the trigger level when the follower exercises the option,  $B$  is a constant that is determined by the value matching. The solution is

$$V_L^1(X) = \frac{R_{11}^1 X_t}{\delta} - \frac{f_1}{r} + \frac{(R_{21}^1 - R_{11}^1) X_L^1}{\delta \nu_1} \left( \frac{X}{X_L^1} \right)^{\nu_1} + \frac{(R_{22}^1 - R_{21}^1) X_F^2}{\delta} \left( \frac{X}{X_F^2} \right)^{\nu_1}, \quad (12)$$

where

$$X_L^1 = X_{LN}^1 = \frac{\nu_1}{\nu_1 - 1} \frac{\delta(f_2 - f_1 + rI)}{r(R_{21}^1 - R_{11}^1)}. \quad (13)$$

The follower's value function must satisfy

$$\frac{1}{2} \sigma^2 X^2 V_{XX}^2 + (r - \delta) X V_X^2 - r V^2 + X R_{11}^2 - f_1 = 0,$$

with the boundary conditions

$$\begin{aligned} V^2(X_F^2) &= \frac{R_{22}^2 X_F^2}{\delta} - \frac{f_2}{r} - \rho I, \\ V_X^2(X_F^2) &= \frac{R_{22}^2}{\delta}, \\ V^2(X_L^1) &= \frac{R_{21}^2 X_L^1}{\delta} - \frac{f_1}{r} + D (X_L^1)^{\nu_1}, \end{aligned} \quad (14)$$

where  $D$  and  $C$  are constants. This results in the follower's value function

$$V_F^2(X) = \frac{R_{11}^2 X_t}{\delta} - \frac{f_1}{r} + \frac{(R_{21}^2 - R_{11}^2) X_L^1}{\delta} \left( \frac{X}{X_L^1} \right)^{\nu_1} + \frac{(R_{22}^2 - R_{21}^2) X_F^2}{\delta \nu_1} \left( \frac{X}{X_F^2} \right)^{\nu_1}, \quad (15)$$

where

$$X_F^2 = \frac{\nu_1}{\nu_1 - 1} \frac{\delta(f_2 - f_1 + r\rho I)}{r(R_{22}^2 - R_{21}^2)}.$$

*ii) Preemptive leader follower:* In this case firm 1 chooses  $X_L^1$  that does not satisfy the smooth pasting condition (11), and the corresponding firm value becomes

$$\begin{aligned} V^1(X) = & \frac{R_{11}^1}{\delta} X - \frac{f_1}{r} + \frac{(R_{22}^1 - R_{21}^1)X_F^2}{\delta} \left( \frac{X}{X_F^2} \right)^{\nu_1} \\ & + \left[ \frac{(R_{21}^1 - R_{11}^1)X_L^1}{\delta} - \frac{f_2 - f_1 + rI}{r} \right] \left( \frac{X}{X_L^1} \right)^{\nu_1}. \end{aligned} \quad (16)$$

*iii) Simultaneous move:* If both firms exercise simultaneously at the trigger level  $X_S^i$  the corresponding value functions are given by

$$V_S^i(X) = \frac{R_{11}^i}{\delta} X - \frac{f_1}{r} + \frac{(f_2 - f_1 + rI^i)}{r(\nu_1 - 1)} \left( \frac{X}{X_S^i} \right)^{\nu_1} \quad (17)$$

and the trigger levels

$$X_S^i = \frac{\nu_1}{\nu_1 - 1} \frac{\delta(f_2 - f_1 + rI^i)}{r(R_{22}^i - R_{11}^i)}.$$

*iv) Off-equilibrium, firm 2 leads:* If firm 2 does not act as the follower and instead exercises as the leader its value function is

$$\begin{aligned} V_L^2(X) = & \frac{R_{11}^2}{\delta} X - \frac{f_1}{r} + \frac{(R_{12}^2 - R_{11}^2)X_L^2}{\delta\nu_1} \left( \frac{X}{X_L^2} \right)^{\nu_1} \\ & + \frac{(R_{22}^2 - R_{12}^2)X_F^1}{\delta} \left( \frac{X}{X_F^1} \right)^{\nu_1} \end{aligned} \quad (18)$$

with  $X_L^2$  as the leader's trigger for firm 2 and  $X_F^1$  the follower's trigger for

firm 1.

**Part 2: Conditions for simultaneous equilibrium:** Since  $I^1 = I < \rho I = I^2$  the only candidate for a simultaneous equilibrium is trigger level  $X_S^1 < X_S^2$ . For a simultaneous equilibrium to occur (i) the value of firm 1 being the leader has to be smaller than moving simultaneously with firm 2, and (ii) firm 2 has to find it profitable to move simultaneously with firm 1 and not to wait and act as the follower. The difference between the firm 1 leader value assuming immediate exercise and the value from waiting for simultaneous exercise is given by

$$\Delta(X, \rho) = \frac{(R_{21}^1 - R_{11}^1)X}{\delta} - \frac{f_2 - f_1 + rI}{r} + \frac{(R_{22}^1 - R_{21}^1)X_F^2}{\delta} \left(\frac{X}{X_F^2}\right)^{\nu_1} - \frac{(R_{22}^1 - R_{11}^1)X_S^1}{\delta\nu_1} \left(\frac{X}{X_S^1}\right)^{\nu_1}.$$

Note that the leader's value function immediately after capacity expansion is increasing (in the relevant range) and strictly concave in  $X$  while the value function for simultaneous exercise is strictly convex. It can be shown that there exists  $X^{**}$  and  $\rho^{**}$  such that  $\Delta(X^{**}, \rho^{**}) = 0$  and  $\Delta_X(X^{**}, \rho^{**}) = 0$ . Pawlina and Kort (2006) show that this implies for all  $\rho < \rho^{**}$  that both conditions (i) and (ii) hold and a simultaneous equilibrium exists. In case  $\rho^{**} < 1$  a simultaneous equilibrium is not possible.

**Part 3: Conditions for preemptive equilibrium:** In the preceding analysis we have shown that if the investment costs are not too different equilibrium play results in simultaneous investment. Now we focus on identifying a cost difference that does make it unattractive for firm 2 to act as the leader. The value function of firm 2 when it acts as the follower is given

by (15) and when it acts as the leader is given by (18). In a preemptive equilibrium, firm 1 is forced to expand just before the first instant at which firm 2 would be indifferent between acting as the leader or acting as the follower with firm 1 exercising immediately. The difference between firm 2's value function as a leader and its value as a follower after a preemptive expansion by the leader is

$$G(X, \rho) \equiv \frac{(R_{12}^2 - R_{21}^2)X}{\delta} - \frac{(f_2 - f_1 + \rho r I)}{r} + \frac{(R_{22}^2 - R_{12}^2)X_F^1}{\delta} \left(\frac{X}{X_F^1}\right)^{\nu_1} - \frac{(R_{22}^2 - R_{21}^2)X_F^2}{\delta \nu_1} \left(\frac{X}{X_F^2}\right)^{\nu_1} = V_L^2(X) - V_F^2(X),$$

where  $X_F^1$  is the trigger level when firm 1 acts as the follower. Firm 2 does not have an incentive to become the leader if and only if  $G(X, \rho) \leq 0$  for all  $X \leq X_F^2$ . Given the strict concavity of the value function when firm 2 acts as the leader (18) and the strict convexity when it acts as the follower (15), we need to identify a set of parameters such the convex followers value function is tangent to the concave leader's value function. This holds true if only if we find  $(X^*, \rho^*)$  such that

$$G(X^*, \rho^*) = 0, \tag{19}$$

$$\frac{\partial G(X^*, \rho^*)}{\partial X} = 0. \tag{20}$$

It is straight forward to show that (19) and (20) are satisfied if and only if

$$X^* = \frac{\nu_1}{\nu_1 - 1} \frac{\delta(f_2 - f_1 + r\rho^* I)}{r[R_{12}^2 - R_{21}^2]}. \tag{21}$$

Equations (21) and (19) identify  $\rho^*$ . At the point  $(X^*, \rho^*)$  the value functions of firm 2 acting as the follower is tangent to the value function of firm 2 acting as the leader. Hence, for all  $\rho \geq \rho^*$  the follower does not have an incentive to become the leader and the equilibrium outcome is that firm 1 acts as the leader and firm 2 acts as the follower. This implies a non-preemptive sequential equilibrium as the unique pure strategy MPE.

For  $\rho < \rho^*$  firm 2 has an incentive to become the leader. This incentive exists for all values of  $X$  in the interval  $[X_{LP}^1, X_F^2]$ , where  $X_{LP}^1$  is defined by

$$G(X_{LP}^1, \rho) = 0,$$

i.e., the value at which firm 2 is indifferent between being the leader or being the follower. If the leader's investment trigger satisfies  $X_L^1 < X_{LP}^1$  the follower value of firm 2 exceeds its leader value and firm 2 does not have an incentive to change its follower role. If, however,  $X_L^1 > X_{LP}^1$  the follower has an incentive to preempt the leader marginally which in turn causes the leader to choose  $X_{LP}^1$  as the investment trigger. With this trigger firm 1 acts as the leader while it is optimal for firm 2 to act as the follower.

**Part 4: Random leader follower equilibrium:** If  $\rho = 1$  and  $\rho^{**} < 1$ , none of the pure strategy equilibria above are possible. To define a mixed strategy equilibrium requires an expansion of the strategy space following Fudenberg and Tirole (1985) and Huisman and Kort (1999). In such a mixed strategy equilibrium the leader is chosen randomly at the trigger level  $X_{LP}^1$  while the other firm acts as the follower.

## A.5 Proof of Proposition Five

The leader's value function in case of a non-preemptive equilibrium is given by (12) while in case of a preemptive equilibrium when the expansion trigger does not satisfy the smooth pasting condition it is given by (16). In both cases the follower's value function is given by (15).

## A.6 Proof of Proposition Six

In the case where both firms simultaneously invest their value functions are given by (17).

## A.7 Proof of Proposition Seven

Follows immediately from the firms' value functions and the definition of beta.

## A.8 Proof of Proposition Eight

The value function of firm 2 acting as a follower is

$$V_F^2(K_t, X_t) = \begin{cases} \frac{R_{01}^2 X_t}{\delta} - \frac{f_1}{r} + \frac{f_1 - f_0 + r\rho S}{r(1-\nu_2)} \left(\frac{X_t}{X_C^2}\right)^{\nu_2} & X_t \geq X_C^2 \\ \frac{R_{00}^2 X_t}{\delta} - \frac{f_0}{r} + \rho S & X_t \leq X_C^2, \end{cases} \quad (22)$$

where

$$X_C^2 = \frac{\nu_2 \delta (f_1 - f_0 + r\rho S)}{(1 - \nu_2) r [R_{00}^2 - R_{01}^2]} \quad (23)$$

is the contraction trigger when firm 2 acts as the follower. Now assume, instead, that firm 2 having the smaller salvage value acts as the leader. The

value function of firm 2 at the time when it contracts as the leader is

$$V_L^2(K_t, X_t) = \begin{cases} \frac{R_{10}^2 X_t}{\delta} - \frac{f_0}{r} + \rho S + \frac{X_C^{1,F} [R_{00}^2 - R_{10}^2]}{\delta} \left( \frac{X_t}{X_C^2} \right)^{\nu_2} & X_t \geq X_C^{1,F} \\ \frac{R_{00}^2 X_t}{\delta} - \frac{f_0}{r} + \rho S & X_t \leq X_C^{1,F}, \end{cases} \quad (24)$$

where

$$X_C^{1,F} = \frac{\nu_2 \delta (f_1 - f_0 + rS)}{(1 - \nu_2) r [R_{00}^1 - R_{10}^1]}. \quad (25)$$

is the trigger when firm 1 contracts as the follower and firm 2 acts as the leader. Given our assumptions on the revenue functions it follows that trigger (25) is strictly greater than trigger (23) for  $0 < \rho < 1$ . From this property and the value functions (22) and (24) it can be shown that

$$G(X_t, \rho) \equiv V_L^2(X_t, \rho) - V_F^2(X_t, \rho) \leq 0$$

holds for all  $X_t$ . Hence, firm 2 never has an incentive to become the leader. Therefore sequential exercise of contraction options is the unique pure strategy MPE  $\rho < 1$ . In case of  $\rho = 1$  and the two firms have symmetric salvage values we can use Proposition 3 and 4 in Murto (2004) that establishes the existence of two pure strategy MPE.

## A.9 Proof of Propositions Nine

The derivation of the leader and the follower value functions follows the same arguments used in Proposition 4 with the difference that instead of a call the contraction option corresponds to a put option. The leader's value

function satisfies

$$\frac{1}{2}\sigma^2 X^2 V_{XX}^1 + (r - \delta)XV_X^1 - rV^1 + XR_{11}^1 - f_1 = 0,$$

with the boundary conditions

$$\begin{aligned} V^1(X_C^1) &= \frac{R_{01}^1 X_C^1}{\delta} - \frac{f_0}{r} + S + B (X_C^1)^{\nu_2}, \\ V_X^1(X_C^1) &= \frac{R_{01}^1}{\delta} + B\nu_2 (X_C^1)^{\nu_2-1}, \\ V^1(X_C^2) &= \frac{R_{00}^1 X_C^2}{\delta} - \frac{f_0}{r}, \end{aligned}$$

resulting in the value function stated in the proposition. The leader's contraction trigger is given by

$$X_C^1 = \frac{\nu_2 \delta (f_1 - f_0 + rS)}{(1 - \nu_2)r[R_{01}^1 - R_{11}^1]}.$$

The follower's value function satisfies

$$\frac{1}{2}\sigma^2 X^2 V_{XX}^2 + (r - \delta)XV_X^2 - rV^2 + XR_{11}^2 - f_1 = 0,$$

with the boundary conditions

$$\begin{aligned} V^2(X_C^2) &= \frac{R_{00}^2 X_C^2}{\delta} - \frac{f_0}{r} + \rho S, \\ V_X^2(X_C^2) &= \frac{R_{00}^2}{\delta}, \\ V^2(X_C^1) &= \frac{R_{01}^2 X_C^1}{\delta} - \frac{f_1}{r} + D (X_C^1)^{\nu_2}, \end{aligned}$$

that results in the value function stated in the proposition. The follower's trigger is given by

$$X_C^2 = \frac{\nu_2 \delta (f_1 - f_0 + r\rho S)}{(1 - \nu_2)r[R_{00}^2 - R_{01}^2]}.$$

#### **A.10 Proof of Proposition Ten**

Follows immediately by using the value functions stated in Proposition 9 and the the definition of beta.

## References

- [1] Aguerrevere, F. , 2003, Equilibrium investment strategies and output price behavior: a real option approach. *Review of Financial Studies* 16, 1239-1272.
- [2] Aguerrevere, F. , 2009, Real options, product market competition, and asset returns. *Journal of Finance* 64, 957-983.
- [3] Back, K., and D. Paulson, 2009, Open-loop equilibria and perfect competition in option exercise games. Working Paper.
- [4] Berk, J., R. Green, and V. Naik, 1999, Optimal investment, growth options, and asset returns. *Journal of Finance* 54, 1553 - 1607.
- [5] Boyer, M., P. Lasserre, T. Mariotti, and M. Moreaux, 2001, Real options, preemption, and the dynamics of industry investments. Working Paper.
- [6] Brealey, R. and S. Myers, *Principles of Corporate Finance*. McGraw Hill, 2001.
- [7] Brennan, M., and E. Schwartz, 1985, Evaluating natural resource investments. *Journal of Business* 58, 135-157.
- [8] Carlson, M., A. Fisher, and R. Giammarino, 2004, Corporate investment and asset price dynamics: implications for the cross-section of returns. *Journal of Finance* 59, 2577-2603.

- [9] Carlson, M., A. Fisher, and R. Giammarino, 2006, Corporate investment and asset price dynamics: implications for SEO event studies and long run performance. *Journal of Finance* 61, 1009-1034.
- [10] Carlson, M., A. Fisher, and R. Giammarino, 2008, SEOs, real options, and risk dynamics: empirical evidence. Working paper.
- [11] Cooper, I., 2006, Asset price implications of non-convex adjustment costs of investment. *Journal of Finance*.
- [12] Dixit, A., and R. Pindyck, 1994, *Investment under uncertainty*. Princeton University Press, Princeton.
- [13] Fudenberg, D. and J. Tirole, 1985, Preemption and rent equalization in the adoption of new technology. *Review of Economic Studies*.
- [14] Gomes, J., L. Kogan, and L. Zhang, 2003, Equilibrium cross-section of returns. *Journal of Political Economy* 111, 693-732.
- [15] Grenadier, S., 2002, Option exercise games: an application to the equilibrium investment strategies of firms. *Review of Financial Studies* 15, 691-721.
- [16] Hackbarth, D., and E. Morellec, 2008, Stock returns in mergers and acquisitions. *Journal of Finance* 63, 1213-1252.
- [17] Huisman, K., and P. Kort, 1999, Effects of strategic interaction on the option value of waiting. Working paper.
- [18] Johnson, T., 2002, Rational momentum effects, *Journal of Finance* 57, 585-608.

- [19] Kogan, L., 2004, Asset prices and real investment. *Journal of Financial Economics*, 73, 411 - 431.
- [20] Kuehn, L., 2008, Asset pricing with real investment commitment. Working paper.
- [21] Mason, R, and H. Weeds, 2008, Investment, uncertainty and pre-emption. Working paper.
- [22] McDonald, R., and D. Siegel, 1985, Investment and the valuation of firms when there is an option to shut down. *International Economic Review* 26, 331-49.
- [23] McDonald, R., and D. Siegel, 1986, The value of waiting to invest. *Quarterly Journal of Economics* 101, 707-727.
- [24] Murto, P., 2004, Exit in duopoly under uncertainty. *Rand Journal of Economics* 35, 111-127.
- [25] Novy-Marx, R., 2008, Competition, productivity, and the cross-section of expected returns. Working paper.
- [26] Pastor, L., and P. Veronesi, 2008, Technological revolutions and stock prices. *American Economic Review*, forthcoming.
- [27] Pawlina, G., and P. Kort, 2006, Real options in an asymmetric duopoly: who benefits from your competitive disadvantage? *Journal of Economics and Management Strategy* 15, 1-35.
- [28] Ross, S., J. Westerfield, and J. Jaffe, *Corporate Finance*. Irwin, 1996.

- [29] Sagi, J., and M. Seasholes, 2007, Firm-specific attributes and the cross-section of momentum. *Journal of Financial Economics* 84, 389-434.
- [30] Smets, R., 1991, Exporting vs. FDI: the effect of uncertainty. Working paper, Yale University.
- [31] Thijssen, J., K. Huisman, and P. Kort, 2002, Symmetric equilibrium strategies in game theoretic real option models. Working paper.
- [32] Zhang, L., 2005, The value premium. *Journal of Finance* 60, 67-103.